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A fresh look at neutral meson mixing

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Abstract

In this work we show that the existence of a complete biorthonormal set of eigenvectors of the effective Hamiltonian governing the time evolution of neutral meson system is a necessary condition for diagonalizability of such a Hamiltonian. We also study the possibility of probing the *CPT* invariance by observing the time dependence of cascade decays of type $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$ by employing such basis and exactly determine the *CPT* violation parameter by comparing the time dependence of the cascade decays of tagged P° and tagged $\bar{P}^\circ$.

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1. Introduction

In the Wigner–Weisskopf (W–W) approximation [1] the effective Hamiltonian which describes the P°–$\bar{P}^\circ$ system is not Hermitian. Therefore the eigenkets of this Hamiltonian are indistinguishable (unless the Hamiltonian is normal, $[\hat{H}, \hat{H}^\dagger] = 0$). The reason is that for such a system the orthogonality and completeness relations could not be written in terms of its eigenkets. In the presence of T violation in the P°–$\bar{P}^\circ$ system, we are dealing with a non-Hermitian Hamiltonian which is not normal. Therefore we can use the principles of non-Hermitian quantum mechanics and reconsider the definition of diagonalizability of an operator. We use the biorthonormal basis for this propose. Here we emphasise that when a non-Hermitian and non-normal operator is encountered, use of a complete biorthonormal basis is in order which reduces to orthogonal basis as soon as the operator is considered Hermitian and normal. Therefore we conclude that we may use such a set of basis to describe the time evolution of neutral mesons in the presence of T violation.

As mentioned, in the presence of T violation the eigenkets of a non-Hermitian Hamiltonian does not satisfy the completeness and orthogonality relations. Therefore the eigenkets of such a Hamiltonian are not distinguishable. Due to this fact in writing down the transition amplitudes for cascade decays $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$, we use the biorthonormal basis when the intermediate states are eigenstates of $\hat{H}$.

We prove that the existence of a complete biorthonormal set of eigenvectors of $\hat{H}$ is necessary condition for diagonalizability of the effective Hamiltonian governing the time evolution of the neutral meson systems and write down the spectral form of the Hamiltonian operator with this basis in Section 2. In Section 3 we discuss the time evolution of neutral meson system and introduce the T and *CPT* violation complex parameters and obtain the time evolution of flavor eigenkets. Finally in the last section we study the possibility of probing *CPT* invariance by observation of the time dependence of the cascade decays of type $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$ by using the biorthonormal basis and introduce new ratios of decay amplitudes and exactly determine the *CPT* violation parameter by comparing the time dependence of the cascade decays of tagged P° and tagged $\bar{P}^\circ$.

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2. Diagonalizability and the complete set of biorthonormal basis

A linear operator $\hat{H}$ acting in a separable Hilbert space and having a discrete spectrum is diagonalizable if and only if there are eigenvectors $|\psi_n\rangle$ of $\hat{H}$ and $|\phi_n\rangle$ of $\hat{H}^\dagger$ that form a complete set of biorthonormal basis of $\{\psi_n, \phi_n\}$, i.e. they satisfy

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle, \qquad \hat{H}^\dagger|\phi_n\rangle = E_n^*|\phi_n\rangle, \tag{1}$$

and

$$\langle\psi_m|\phi_n\rangle = \delta_{mn}, \qquad \sum_n |\psi_n\rangle\langle\phi_n| = \sum_n |\phi_n\rangle\langle\psi_n| = 1, \tag{2}$$

where n is the spectral label and † and * denote the adjoint and complex-conjugate respectively as usual. Moreover δ_{mn} is the Kronecker delta function and 1 represents the identity operator.

Nowhere in this definition it is assumed that the operator is normal, i.e., $[\hat{H}, \hat{H}^\dagger] = 0$. A normal operator, in finite dimensions with no extra conditions and in infinite dimensions with appropriate extra conditions, admits a diagonal matrix representation in some orthogonal basis. This is usually called *diagonalizability* by a unitary transformation. In view of (1) and (2) the spectral form of $\hat{H}$ and $\hat{H}^\dagger$ may be written in the following form

$$\hat{H} = \sum_n E_n|\psi_n\rangle\langle\phi_n|, \qquad \hat{H}^\dagger = \sum_n E_n^*|\phi_n\rangle\langle\psi_n|. \tag{3}$$

In order to see the equivalence of the existence of a complete biorthonormal set of eigenvectors of $\hat{H}$ and its diagonalizability, we note that by definition a diagonalizable Hamiltonian $\hat{H}$ satisfies $\hat{A}^{-1}\hat{H}\hat{A} = \hat{H}_\circ$ for an invertible linear operator $\hat{A}$ and a diagonal linear operator $\hat{H}_\circ$, i.e., there is an orthogonal basis $\{|n\rangle\}$ in the Hilbert space and complex numbers E_n such that $\hat{H}_\circ = \sum_n E_n|n\rangle\langle n|$. Then letting $|\psi_n\rangle := \hat{A}|n\rangle$ and $|\phi_n\rangle := (\hat{A}^{-1})^\dagger|n\rangle$, we can easily check that $\{|\psi_n\rangle, |\phi_n\rangle\}$ is a complete biorthonormal system for $\hat{H}$. The converse is also true, for if such a system exists we may set $\hat{A} := \sum_n |\psi_n\rangle\langle n|$ for some orthogonal basis $\{|n\rangle\}$ and by using Eq. (2) check that $\hat{A}^{-1} = \sum_n |n\rangle\langle\phi_n|$ and $\hat{A}^{-1}\hat{H}\hat{A} = \hat{H}_\circ$, i.e., $\hat{H}$ is diagonalizable.

As long as T is invariant (no violation), the effective Hamiltonian is normal. In such a case the orthonormality relations between the basis of $\hat{H}$ are valid and moreover that the eigenkets of $\hat{H}$ are discriminant and the biorthonormal basis turns into orthonormal basis automatically and $|\psi_n\rangle$'s are the same as $|\phi_n\rangle$'s.

3. The time evolution of neutral meson system

In the Wigner–Weisskopf (W–W) approximation, which we shall use throughout, a beam of oscillating and decaying neutral meson system is described in its rest frame by a two component wave function

$$\big|\psi(t)\big\rangle = \psi_1(t)|P^\circ\rangle + \psi_2(t)|\bar{P}^\circ\rangle, \tag{4}$$

where t is the proper time and $|P^\circ\rangle$ stands for K, D, B_d or B_s. The wave function evolves according to a Schrödinger like equation

$$i\frac{d}{dt}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix} = \begin{pmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{pmatrix}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix}. \tag{5}$$

The matrix $\hat{H}$ is usually written as $\hat{H} = \hat{M} - i\hat{\Gamma}/2$, where $\hat{M} = \hat{M}^\dagger$, and $\hat{\Gamma} = \hat{\Gamma}^\dagger$ are 2×2 matrices called the mass and the decay matrices [1–7]. Decomposition of $\hat{H}$ reads

$$\begin{aligned}\hat{H} = {} & |P^\circ\rangle H_{11}\langle P^\circ| + |P^\circ\rangle H_{12}\langle\bar{P}^\circ| \\ & + |\bar{P}^\circ\rangle H_{21}\langle P^\circ| + |\bar{P}^\circ\rangle H_{22}\langle\bar{P}^\circ|.\end{aligned} \tag{6}$$

The flavor basis $\{|P^\circ\rangle, |\bar{P}^\circ\rangle\}$ satisfies orthogonality and completeness relations

$$\begin{aligned}&\langle P^\circ|\bar{P}^\circ\rangle = \langle\bar{P}^\circ|P^\circ\rangle = 0, \qquad \langle P^\circ|P^\circ\rangle = \langle\bar{P}^\circ|\bar{P}^\circ\rangle = 1,\\ &|P^\circ\rangle\langle P^\circ| + |\bar{P}^\circ\rangle\langle\bar{P}^\circ| = 1.\end{aligned} \tag{7}$$

It is readily shown that $\hat{H}$ is not Hermitian. If $[\hat{H}, \hat{H}^\dagger] \neq 0$, then the orthogonality and completeness relations for eigenstates of $\hat{H}$ are not satisfied, i.e., the eigenstates of $\hat{H}$ are not discriminant states. Therefore we cannot diagonalize $\hat{H}$ or write its spectral form though its basis. To do this job we make the benefit of biorthonormal basis. Such basis could be set up for neutral meson system. Indeed the Hamiltonian is diagonalizable only with such a set of basis.

According to Section 2 the eigenvalues of $\hat{H}$ are denoted by $\mu_a = m_a - i\Gamma_a/2$ and $\mu_b = m_b - i\Gamma_b/2$ corresponding to the eigenvectors $|P_a\rangle$ and $|P_b\rangle$ respectively. So that

$$\begin{aligned}&\hat{H}|P_a\rangle = \mu_a|P_a\rangle,\\ &\hat{H}|P_b\rangle = \mu_b|P_b\rangle.\end{aligned} \tag{8}$$

We also denote the eigenvalues of $\hat{H}^\dagger$ by $\mu_a^* = m_a + i\Gamma_a/2$ and $\mu_b^* = m_b + i\Gamma_b/2$ corresponding to the eigenvectors $|\tilde{P}_a\rangle$ and $|\tilde{P}_b\rangle$ respectively. So that

$$\begin{aligned}&\hat{H}^\dagger|\tilde{P}_a\rangle = \mu_a^*|\tilde{P}_a\rangle,\\ &\hat{H}^\dagger|\tilde{P}_b\rangle = \mu_b^*|\tilde{P}_b\rangle.\end{aligned} \tag{9}$$

It is not difficult to check that the set $\{|P_n\rangle, |\tilde{P}_n\rangle\}$, $n = a, b$, is a complete biorthonormal system for $\hat{H}$ such that

$$\langle P_a|\tilde{P}_b\rangle = \langle\tilde{P}_a|P_b\rangle = 0, \qquad \langle P_a|\tilde{P}_a\rangle = \langle\tilde{P}_b|P_b\rangle = 1, \tag{10}$$

$$|P_a\rangle\langle\tilde{P}_a| + |P_b\rangle\langle\tilde{P}_b| = |\tilde{P}_a\rangle\langle P_a| + |\tilde{P}_b\rangle\langle P_b| = 1. \tag{11}$$

According to the definition given in the previous section, $\hat{H}$ could be diagonalized such that

$$\chi^{-1}\hat{H}\chi = \begin{pmatrix}\mu_a & 0\\ 0 & \mu_b\end{pmatrix}, \qquad \chi = \begin{pmatrix}p_a & q_a\\ p_b & -q_b\end{pmatrix}, \tag{12}$$

which means

$$\begin{aligned}&|P_a\rangle = p_a|P^\circ\rangle + q_a|\bar{P}^\circ\rangle,\\ &|P_b\rangle = p_b|P^\circ\rangle - q_b|\bar{P}^\circ\rangle,\end{aligned} \tag{13}$$

and