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# Perturbative aspects and conformal solutions of $F(R)$ gravity

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**Abstract**

We investigate perturbative aspects of gravity with a general $F(R)$ Lagrangian, as well as nonperturbative dilatonic solutions. For the first part, we are interested in stability and the definition of asymptotic charges. The main result of this study is that, while generic $F(R)$ theories are stable under metric perturbations, they are expected to show instabilities against curvature perturbations when the Lagrangian includes $1/R$ terms. For the second part, one is interested on exact solutions, and we explicitly construct kink-like solutions of the Liouville type for the dilaton field for $F(R)$ having the form $R + \gamma R^n$, in two and in four dimensions.


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## 1. Introduction

In this work we investigate perturbative and nonperturbative effects in gravity, motivated by the cosmological constant problem [1]. As one knows, the cosmological constant engenders an important but very hard problem to solve due to both cosmological and phenomenological reasons. In the cosmic evolution of the Universe, particle physics introduces the main mechanisms to govern the several phases of the Universe, and cosmology drives the way the cosmological constant is accounted for in each phase. However, the presence of spontaneous symmetry breaking, which is the basic mechanism underlying unification of the fundamental interactions in nature, inevitably injects vacuum energy density even if no such term is present in the theory. For this reason, one realizes that not only the physics at the Planck scale, but also the low energy infrared dynamics of gravity itself is essential to the resolution of the cosmological constant problem. The reasoning establishes direct connection between nonperturbative effects in gravity and the cosmological constant problem.

Nowadays, the cosmological constant problem must include the important discovery that the Universe is currently evolving in an accelerating phase [2]. There are several distinct possibilities of taking acceleration into account: from one side, we can keep standard geometry, incorporating modifications on the matter contents of the model. Interesting investigations which deal with this possibility include, for instance, the cosmological constant [1], dynamical scalar fields [3], Chaplygin fluid [4], and phantom dynamics [5]. Another possibility deals with modifications of geometry, changing General Relativity; here an important class of theories includes models which depend on the Ricci scalar, usually named $F(R)$ theories [6]. In the present work, we will deal with the latter, that is, we will investigate extended gravity, changing $R \to F(R)$, for $F(R) = R + \gamma R^n$, including the cases $n = 2$ and $n = -1$. This modification brings interesting novelties, as we will show below. The motivation goes beyond the recent discovery of the accelerated expansion of the Universe [2], and it is very natural to expect that, due to the fact that the curvature of the present Universe is very small, the nonperturbative effects should be characteristic for some typical forms of $F(R)$. In particular,

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the interest in $n=2$ also appear from semi-classical investigations, and the case of $F(R)=\exp(-\alpha R)$ suggests the presence of instanton effects.

The investigation of nonperturbative solutions is of direct interest to a diversity of applications, including black hole, integrability, noncommutative geometry and other problems [9,10]. The nonperturbative issues are in general hard to solve, mainly in high energy physics in curved space–time. For this reason, we are planning to approach the problem with the use of the nonperturbative procedure referred to as the first-order formalism, in which one solves equations of motion with solutions of first-order differential equations. Some of us have recently done some progress in extending the formalism to curved space–time [11], and we now consider this possibility in connection with dilaton gravity.

In the present study we organize this Letter as follows. In the next Section 2 we derive the equations of motion for $F(R)$ gravity theories and the conserved charges, and then we study stability of solutions in these theories. In Section 3 we turn attention to gravity in the conformal sector, and there we describe the $F(R)$ model restricted to the conformal metric, investigating the corresponding equations of motion in two and in four dimensions. We end the work in Section 4, where we include some final considerations.

## 2. Equations of motion and conserved charges

Let us lay down the notation, starting with the $F(R)$ action

$$S=-\int d^Dx\,\sqrt{|g|}F(R) \tag{1}$$

where $F(R)$ is equal to $R+2\Lambda$ for the usual Einstein–Hilbert action with a cosmological constant $\Lambda$. The equations of motion for this theory look like

$$\frac{1}{2}g_{ac}F(R)-F'(R)R_{ac}-F''(R)\big[\nabla_a\nabla_cR-\nabla_b\nabla^bRg_{ac}\big]-F'''(R)\big[\nabla_aR\nabla_cR-g_{ac}\nabla_dR\nabla^dR\big]=0. \tag{2}$$

Let us now compute the conserved charge associated with this Lagrangian, following the general procedure of [12]. From above, the boundary term one obtains from the Lagrangian is

$$\theta^b=-F'(R)\big(\nabla_a\big(\delta g^{ab}\big)-\nabla^b(\delta g)\big)-\nabla_aF'(R)\delta g^{ab}+\nabla^bF'(R)(\delta g) \tag{3}$$

that is to say, the full variation of the action can be written as $\delta\mathcal{L}=\sqrt{|g|}E_{ab}\delta g^{ab}+\sqrt{|g|}\nabla_a\theta^a$, where $E_{ab}$ are the equations of motion (2). Now, the Noether procedure gives us a conserved current associated with a symmetry generated by a vector field $\xi^a$. Those types of diffeomorphisms change the metric as $\delta g_{ab}=\nabla_a\xi_b+\nabla_b\xi_a$. The conserved current is

$$J^a\big[\xi^b\big]=\theta^a-\xi^a\mathcal{L}. \tag{4}$$

The existence of this current stems directly from the proof of the theorem. The hard step in any theory which is invariant under diffeomorphisms is to find the charge associated with $J^a$. Consider the Poincaré dual to $J^a$: $\mathbf{J}_{a_1\ldots a_{n-1}}=\epsilon_{aa_1\ldots a_{n-1}}J^a$, where $\epsilon_{a_1\ldots a_n}$ is the volume form associated with the metric. We will exceptionally use $n$ for the dimension of space–time in this subsection to avoid cluttering the notation. Conservation of the current implies that, *on shell*, the $n-1$ form $\mathbf{J}$ is closed $d\mathbf{J}=0$, so it can be written locally as a exterior derivative: $\mathbf{J}_{a_1\ldots a_{n-1}}=\nabla_{[a_1}\mathbf{Q}_{a_2\ldots a_{n-1}]}=\partial_{[a_1}\mathbf{Q}_{a_2\ldots a_{n-1}]}$, or, analogously with the duals $J^b=\nabla_aQ^{[ab]}$. We can check directly that this equation above implies the conservation of the current

$$\nabla_bJ^b=\nabla_b\nabla_aQ^{[ab]}=-\frac{1}{2}[\nabla_a,\nabla_b]Q^{[ab]}=\frac{1}{2}\big(R_{abc}{}^aQ^{[cb]}+R_{abc}{}^bQ^{[ac]}\big)=\frac{1}{2}\big(-R_{bc}Q^{[cb]}+R_{ac}Q^{[ac]}\big)=R_{ac}Q^{[ac]}=0. \tag{5}$$

If we are able to find such $Q^{ab}$ we can write directly the conserved charge by integrating the corresponding $n-2$ form $\mathbf{Q}$ over a suitable $n-2$ hypersurface. The procedure is then analogous to the definition of conserved charges in field theory which can be written under certain assumptions as the integral of a charge flux over a sphere (a $n-2$ dimensional hypersurface in $\mathbb{R}^{3,1}$) "at infinity".

As it turns out, one can indeed write a $Q^{ab}$ for the theory above; we suppose it has the form $Q^{ab}[\xi^c]=2W^{[a}\xi^{b]}+2X\nabla^{[a}\xi^{b]}$, for some metric-dependent $W^a$ and $X$. Taking its divergence, we will find several terms

$$\nabla_aQ^{ab}=\xi^b\nabla_aW^a-\xi^a\nabla_aW^b+W^a\nabla_a\xi^b-W^b\nabla_a\xi^a+\nabla_aX\big(\nabla^a\xi^b-\nabla^b\xi^a\big)+X\big(\nabla_a\nabla^a\xi^b-\nabla_a\nabla^b\xi^a\big). \tag{6}$$

On the other hand, the direct calculation from (4) gives

$$\begin{aligned}J^b={}&F'(R)\big[\nabla_a\big(\nabla^a\xi^b+\nabla^b\xi^a\big)-2\nabla^b\nabla_a\xi^a\big]-F''(R)\nabla_aR\big(\nabla^a\xi^b+\nabla^b\xi^a-2g^{ab}\nabla_c\xi^c\big)\\&-2F'(R)\nabla^bR\nabla_a\xi^a+\xi^bF(R).\end{aligned} \tag{7}$$

Comparing the terms involving two derivatives of $\xi^a$, we find that $X$ must be $X=F'(R)$, up to some term proportional to the equations of motion. The terms involving only one derivative of $\xi^a$

$$W^a\nabla_a\xi^b-W^b\nabla_a\xi^a+\nabla_aX\big(\nabla^a\xi^b-\nabla^b\xi^a\big)\stackrel{?}{=}-F''(R)\nabla_aR\big[\nabla^a\xi^b+\nabla^b\xi^a-2g^{ab}\nabla_c\xi^c\big] \tag{8}$$