Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 649 (2007) 454–462
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Quantum effects in softly broken gauge theories in curved space–times

I.L. Buchbinder [a,b], G. de Berredo-Peixoto [b], I.L. Shapiro [b,*,1]

[a] Department of Theoretical Physics, Tomsk State Pedagogical University, 634041 Tomsk, Russia
[b] Departamento de Física – ICE, Universidade Federal de Juiz de Fora Juiz de Fora, CEP: 36036-330, MG, Brazil

Received 23 March 2007; received in revised form 18 April 2007; accepted 20 April 2007

Available online 4 May 2007

Editor: M. Cvetič

Abstract

The soft breaking of gauge or other symmetries is the typical Quantum Field Theory phenomenon. In many cases one can apply the Stückelberg procedure, which means introducing some additional field or fields and restore the gauge symmetry. The original softly broken theory corresponds to a particular choice of the gauge fixing condition. In this Letter we use this scheme for performing quantum calculations for some softly broken gauge theories in an arbitrary curved space–time. The following examples are treated in details: Proca field, massive QED and massive torsion coupled to fermion. Furthermore we present a qualitative discussion of the discontinuity of quantum effects in the massive spin-2 field theory, paying special attention to the similarity and differences with the massless limit in the spin-1 case.

PACS: 04.62.+v; 11.10.Gh; 11.15.-q; 11.30.-j

Keywords: Curved space–time; Soft symmetry breaking; Renormalization

1. Introduction

Theories with softly broken gauge symmetries emerge frequently in various branches of Quantum Field Theory (QFT). All attempts to consider the soft breaking of the non-Abelian gauge symmetry met serious difficulties [1], but anyway they represent an important phase in the development of the modern high energy physics. One more example is supersymmetry which must be (most likely softly) broken in order to address the phenomenological applications and eventually experimental tests [2]. Another interesting application of the soft symmetry breaking is the effective QFT approach to the propagating torsion [3–5]. The most relevant completely antisymmetric component of torsion can be described by the dual axial vector coupled to fermions through the axial vector current. The presence of the symmetry breaking mass of this axial vector is necessary for the consistency of the effective theory in the low-energy sector. Furthermore one can mention an important problem of discontinuity in the massless limit for the massive spin-2 (sometimes called massive graviton) field [6]. The massless and massive spin-2 particles have different number of degrees of freedom even if the mass is extremely small, hence there is no smooth massless limit, e.g., in the gravitational interaction. The problem of classical discontinuity can be solved if, instead of the flat background, one takes the curved one [7], which in this case must be dS or AdS space. However, according to the publications [8], the discontinuity persists in the quantum corrections, even in curved space. The last two examples show the importance of evaluating the quantum corrections in the theories with softly broken symmetries, especially in curved space–time.

In the mentioned cases one is interested not only in the classical aspects of the theory, but also in deriving quantum corrections. The subject of the present Letter is the calculation of effective action for softly broken gauge theories in curved space–time. In this

* Corresponding author.
E-mail addresses: joseph@tspu.edu.ru (I.L. Buchbinder), guilherme@fisica.ufjf.br (G. de Berredo-Peixoto), shapiro@fisica.ufjf.br (I.L. Shapiro).
1 Also at Tomsk State Pedagogical University, Russia.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.04.039

case a kinetic term and interactions of classical action are gauge invariant while the massive terms are not. As a result the standard methods for evaluating effective action face serious technical difficulties. Our general strategy will be as follows: in each case we shall apply the Stückelberg procedure [9], that is, restore the gauge symmetry by introducing an extra field or a set of fields.

The simplest example is the Proca field model in curved space, which is considered here as a kind of simple pedagogical example, illustrating the method. The restoration of gauge symmetry requires introducing a new scalar field. Then, the original Proca theory corresponds to the special gauge fixing in a new theory, while the quantum calculations are performed in some different gauge, which is most useful from technical viewpoint. Let us notice that the gauge fixing dependence of the effective action should vanish on shell. The situation is especially simple for the one-loop corrections, because in this case the difference between the effective actions calculated in different gauges is proportional to the classical equations of motion. Therefore, when evaluating quantum corrections to the vacuum action (that is the action of external, e.g., gravitational field), the result is gauge fixing independent. If we are dealing with the interacting theory and look also for the renormalization in the matter sector, some additional effort may be requested.

As we shall see in what follows, our approach paves the way for much simpler and more efficient calculation of quantum corrections. The difference is especially explicit for the massive torsion–fermion system which was originally elaborated in [4]. The present method provides an independent verification of our previous result [4] and also enables one to perform the calculations in an arbitrary curved space–time, something that was impossible in the framework used in [4]. The Letter is organized as follows. In Section 2 we consider quantum calculations for the Proca field in curved space. In Section 3 the result is generalized for the massive QED and we also learn some important aspects of dealing with interacting fields. Section 4 is devoted to the massive torsion–fermion system. In Section 5 we discuss the one-loop calculations for the massive spin-2 field, especially focusing on the problem of discontinuity of quantum corrections in the massless limit. Finally, in Section 6 we draw our conclusions.

2. Proca theory in curved space

As a first example, consider the massive Abelian vector fields, which is also called Proca model. The action of the theory in curved space has the form

$$S_P = \int d^4x\,\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}M^2A_\mu^2\right\}. \tag{1}$$

Here and throughout the rest of the Letter we use the condensed notations $F_{\mu\nu}^2 = F_{\mu\nu}F^{\mu\nu}$ and $A_\mu^2 = A_\mu A^\mu$. Also, we consistently disregard irrelevant surface terms. The main problem in deriving the quantum corrections here is the softly broken gauge invariance. As a consequence of that, the bilinear form of the action

$$\hat{H} = H_\alpha^\mu = \delta_\alpha^\mu\Box - \nabla_\alpha\nabla^\mu - R_\alpha^\mu - M^2\delta_\alpha^\mu \tag{2}$$

is degenerate while the theory is not invariant under the standard gauge transformation. The non-invariance does not permit the use of the usual Faddeev–Popov technique for eliminating the degeneracy. The known way of solving this problem [10] requires introducing an auxiliary operator

$$\hat{H}^* = H^{*\nu}_{\ \ \mu} = -\nabla_\mu\nabla^\nu + M^2\delta_\mu^\nu, \tag{3}$$

which satisfies the following two properties:

$$\begin{aligned} &H_\alpha^\mu H^{*\nu}_{\ \ \mu} = M^2\left(\delta_\alpha^\nu\Box - R_\alpha^\mu - M^2\delta_\alpha^\mu\right),\\ &\operatorname{Tr}\ln\hat{H}^* = \operatorname{Tr}\ln\left(\Box - M^2\right). \end{aligned} \tag{4}$$

As a result we arrive at the following relation[2]:

$$-\frac{1}{2}\operatorname{Tr}\ln\hat{H} = -\frac{1}{2}\operatorname{Tr}\ln\left(\delta_\alpha^\nu\Box - R_\alpha^\mu - M^2\delta_\alpha^\mu\right) + \frac{1}{2}\operatorname{Tr}\ln\left(\Box - M^2\right). \tag{5}$$

An obvious advantage of the last formula is that both operators at the r.h.s. are not degenerate and admit a simple use of the standard Schwinger–DeWitt technique for the divergences and even the use of a more advanced method for deriving the non-local terms in the second-order in curvature approximation [12,13]. Looking at the expression (5) one can observe certain similarity with the massless case. In both cases we meet contributions from minimal vector and scalar operators. Indeed, the second contribution in (5) is analogous to the ghost contribution in the massless case, but there is a factor 1 instead of a factor 1/2 in front of the term $\operatorname{Tr}\ln\Box$ in the strictly massless case. As a result of this difference one can observe a discontinuity in the vacuum contribution of massive vector field in the massless limit. The difference between the $M \to 0$ limit in Eq. (5) and the contribution of a massless vector is exactly equal to the contribution of a minimal massless scalar. Which scalar is that?

[2] The one-loop Euclidean effective action is given by the formula $\bar{\Gamma}^{(1)} = -\frac{1}{2}\operatorname{Tr}\ln\hat{H}$ (see, e.g., [10,11]).