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Higher dimensional rotating black holes in Einstein–Maxwell theory with negative cosmological constant

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Abstract

We present arguments for the existence of charged, rotating black holes with equal-magnitude angular momenta in an odd number of dimensions $D \geqslant 5$. These solutions possess a regular horizon of spherical topology and approach asymptotically the anti-de Sitter spacetime background. We analyze their global charges, their gyromagnetic ratio and their horizon properties.

1. Introduction

Recently a tremendous amount of interest has focused on anti-de Sitter (AdS) spacetime. This interest is mainly motivated by the proposed correspondence between physical effects associated with gravitating fields propagating in AdS spacetime and those of a conformal field theory (CFT) on the boundary of AdS spacetime [1,2].

In this context, the black holes with cosmological constant $\Lambda < 0$ are of special interest since they would offer the possibility of studying the non-perturbative structure of some CFTs. Higher dimensional rotating black holes with AdS asymptotics have been studied by various authors, starting with Hawking et al. [3] who generalized the $D = 4$ Kerr–AdS solution to five dimensions with arbitrary angular momenta, and to all dimensions with only a single nonzero angular momentum. The generalization of these solutions to the full set of independent angular momenta was given in [4].

It is of interest to generalize these higher dimensional Kerr–AdS metrics further, by including matter fields. Several exact solutions describing charged rotating black holes have been found recently in gauged supergravities in $D = 5$ [5], and $D = 7$ [6] dimensions. Apart from Abelian fields with a Chern–Simons term, these configurations usually contain scalar fields with a nontrivial scalar potential.

The main purpose of this Letter is to report progress on this problem by presenting numerical evidence for the existence a set of asymptotically AdS charged rotating black holes. These configurations exist in an odd number of dimensions, $D \geqslant 5$. They possess a regular horizon of spherical topology, and their angular momenta are all of equal-magnitude, thus factorizing the angular dependence. The same approach was employed recently to construct asymptotically flat charged rotating black holes in higher dimensions [7,8].

Also, instead of specializing to a particular supergravity model, we shall consider pure Einstein–Maxwell (EM) theory with negative cosmological constant. Although this theory is non-supersymmetric in itself for $D > 4$, it enters all gauged supergravities as the basic building block. Therefore one can expect the basic features of its solutions to be generic.

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The Letter is structured as follows: in Section 2 we present the general framework and analyze the field equations. The boundary conditions and the black hole properties are discussed in Section 3. We present the numerical results in Section 4, and conclude with Section 5, where further applications are addressed.

2. Action and ansatz

We consider the EM action with a negative cosmological constant Λ

$$I=\frac{1}{16\pi G_D}\int_{\mathcal{M}} d^Dx\,\sqrt{-g}\big(R-2\Lambda-F_{\mu\nu}F^{\mu\nu}\big)-\frac{1}{8\pi G_D}\int_{\partial\mathcal{M}} d^{D-1}x\,\sqrt{-h}K, \tag{2.1}$$

where $D=2N+1$ ($N\geqslant 2$), G_D is the D-dimensional Newton constant, R is the curvature scalar, and $F_{\mu\nu}$ is the gauge field strength tensor ($F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, with gauge potential A_μ). The last term in (2.1) is the Hawking–Gibbons surface term [9], which is required in order to have a well-defined variational principle. K is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and h is the induced metric of the boundary. We denote $\Lambda=-(D-2)(D-1)/(2\ell^2)$.

The field equations associated with the action (2.1) are the Einstein equations

$$G_{\mu\nu}+\Lambda g_{\mu\nu}=2\left(F_{\mu\rho}F_\nu{}^\rho-\frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right), \tag{2.2}$$

and the gauge field equations

$$\nabla_\mu F^{\mu\nu}=0. \tag{2.3}$$

We consider stationary black hole space–times with N azimuthal symmetries, representing charged $U(1)$ generalizations of the corresponding set of vacuum solutions discussed in [4]. The symmetries imply the existence of $N+1$ commuting Killing vectors, $\xi\equiv\partial_t$, and $\eta_{(k)}\equiv\partial_{\varphi_k}$, for $k=1,\ldots,N$. While the general EM-AdS black holes then possess N independent angular momenta, we here restrict to black holes with equal-magnitude angular momenta and with spherical horizon topology.[1]

We employ a parametrization for the metric corresponding to a generalization of the ansatz used previously for asymptotically flat solutions [7]. It has the general form

$$\begin{aligned} ds^2={}&-b(r)\,dt^2+\frac{dr^2}{u(r)}+g(r)\sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)d\theta_i^2+p(r)\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\left(\varepsilon_k\,d\varphi_k-\frac{\omega(r)}{r}dt\right)^2\\ &+\big(g(r)-p(r)\big)\left\{\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\,d\varphi_k^2-\left[\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k\,d\varphi_k\right]^2\right\}, \end{aligned} \tag{2.4}$$

where $\theta_0\equiv 0$, $\theta_i\in[0,\pi/2]$ for $i=1,\ldots,N-1$, $\theta_N\equiv\pi/2$, $\varphi_k\in[0,2\pi]$ for $k=1,\ldots,N$, and $\varepsilon_k=\pm1$ denotes the sense of rotation in the kth orthogonal plane of rotation. For such solutions, the isometry group is enhanced from $R\times U(1)^{N+1}$ to $R\times U(N+1)$, where R denotes the time translation. This symmetry enhancement allows us to deal only with ordinary differential equations (ODEs).

The vacuum black holes discussed in [4] are recovered for vanishing gauge field and

$$\begin{aligned} &u(r)=1+\frac{r^2}{\ell^2}-\frac{2\hat{M}\Xi}{r^{D-3}}+\frac{2\hat{M}\hat{a}^2}{r^{D-1}}, \qquad p(r)=r^2\left(1+\frac{2\hat{M}\hat{a}^2}{r^{D-1}}\right),\\ &\omega(r)=\frac{2\hat{M}\hat{a}}{r^{D-4}p(r)}, \qquad g(r)=r^2, \qquad b(r)=\frac{r^2u(r)}{p(r)}, \end{aligned} \tag{2.5}$$

where $\hat{M}$ and $\hat{a}$ are two constants related to the solutions' mass and angular momentum, and $\Xi=1-\hat{a}^2/\ell^2$ (see [11] for a discussion of the basic features of these solutions). For the numerical calculations a convenient parametrization is given by

$$u(r)=\frac{f(r)}{m(r)}\left(\frac{r^2}{\ell^2}+1\right), \qquad b(r)=\left(\frac{r^2}{\ell^2}+1\right)f(r), \qquad g(r)=\frac{m(r)}{f(r)}r^2, \qquad p(r)=\frac{n(r)}{f(r)}r^2. \tag{2.6}$$

The ansatz for the U(1) potential, consistent with the symmetries of the line element (2.4), is given by [7]

$$A_\mu\,dx^\mu=a_0\,dt+a_\varphi\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k\,d\varphi_k. \tag{2.7}$$

[1] Asymptotically AdS rotating charged topological black hole solutions with zero scalar curvature of the event horizon are known in closed form [10], but they were found for a different metric ansatz, and they possess rather different properties.