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Quasinormal modes and phase transition of black holes

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Abstract

We have studied the scalar field as well as the fermionic field perturbations in the background of the massless BTZ black holes. Comparing with the perturbation results in the generic non-rotating BTZ black hole background, we found that the massless BTZ hole contains only normal modes in the perturbations. We argued that this special property reflects that the massless BTZ black hole is a different phase from that of the generic non-rotating BTZ hole.

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Black holes' quasinormal modes (QNM) have been an intriguing subject of discussions in the past decades [1–3]. The QNMs is believed as a characteristic sound of black holes, which describes the damped oscillations under perturbations in the surrounding geometry of a black hole with frequencies and damping times of the oscillations entirely fixed by the black hole parameters. The QNMs of black holes has potential astrophysical interest since it could lead to the direct identification of the black hole existence through gravitational wave observation to be realized in the near future [1,2]. Motivated by the discovery of the AdS/CFT correspondence, the investigation of QNM in anti-de Sitter (AdS) spacetimes became appealing in the past several years. It was argued that the QNMs of AdS black holes have direct interpretation in term of the dual conformal field theory (CFT) [3–9]. Attempts of using QNMs to investigate the dS/CFT correspondence has also been given [10]. Recently QNMs in asymptotically flat spaces have acquired further attention, since the possible connection between the classical vibrations of a black hole spacetime and various quantum aspects was proposed by relating the real part of the QNM frequencies to the Barbero–Immirzi (BI) parameter, a factor introduced by hand in order that loop quantum gravity reproduces correctly the black hole entropy [11]. The extension has been done in the dS background [12], however in the AdS black hole spacetime, the direct relation has not been found [13]. Further motivation of studying the QNMs has been pointed out in a very recent paper arguing that QNMs can reflect the black hole phase transition [14]. By calculating the QNMs of electromagnetic perturbations, in [14] it was claimed that they found the evidence of the phase transition in the QNMs behavior for small topological black holes with scalar hair once disclosed in [15].

The motivation of the present Letter is to further explore the possibility of disclosing the black hole phase transition in its QNMs behavior. We will concentrate our attention on the three-dimensional spacetimes. The mathematical simplicity in the three-dimensional cases can help us to understand the physics better. The non-rotating ($J=0$) BTZ black hole is described by the line element

$$ds^2 = -\left(-M + \frac{r^2}{l^2}\right)dt^2 + \left(-M + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2\, d\phi^2. \tag{1}$$

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It possesses a continuous mass spectrum to the massless AdS black holes ($M = 0$) with different topology

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{l^2}{r^2}dr^2 + r^2\,d\phi^2, \tag{2}$$

where we find a degenerate event horizon at the origin of the coordinate $r = 0$. Cai et al. observed that when $M \to 0$, in the microcanonical ensemble some second moments diverge [16]. The divergence of the second moments means that the fluctuation is very large which breaks down the rigorous meaning of thermodynamical quantities. This is the characteristic of the point of phase transition [17]. The massless BTZ black hole has zero Hawking temperature, zero entropy and vanishing heat capacity, which is the same corresponding to that of the usual extreme holes. For the extreme BTZ black hole, its wave dynamics behavior has been compared to that of the non-extreme BTZ hole in [18]. In [19] it was shown that the massless BTZ black hole persists supersymmetries, while the generic non-rotating BTZ black hole does not, which further manifested that the massless hole and the generic BTZ hole are two different phases. The massless hole is a critical point which separates the generic non-rotating BTZ black hole from the AdS space [20]. We would like to investigate the wave dynamics in the massless BTZ hole background and examine whether the perturbation can help to disclose that it is a different phase from that of the generic non-rotating BTZ hole. To obtain a general and solid result, we will first reexamine the scalar perturbation which was investigated in [21], then we will extend our discussion to the fermionic perturbation of the massless BTZ black hole. Besides the study of the perturbation dynamics in the bulk, we will also study from the CFT side. We will compare our results in the massless BTZ background to those obtained in the non-rotating BTZ hole [6,7].

For the massless BTZ hole, the scalar perturbation can be described by

$$\left[\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right) - \mu_0^2\right]\Phi = 0,$$

where μ_0 is the mass of the scalar field. Using the separation of the wave function $\Phi(t,r,\phi) = \frac{1}{\sqrt{r}}R(r)e^{-i\omega t}e^{im\phi}$, the radial wave equation becomes

$$r^2R''(r) + 2rR'(r) + l^2\left(\frac{l^2\omega^2 - m^2}{r^2} - \mu_0^2 - \frac{3}{4l^2}\right)R(r) = 0. \tag{3}$$

The solution of the radial equation is given as a linear combination

$$R(r) = AR^{(1)}(r) + BR^{(2)}(r), \tag{4}$$

where the functions $R^{(1)}(r)$ and $R^{(2)}(r)$ read

$$R^{(1)} = \left(\frac{1}{r}\right)^{\frac{1}{2}+\beta} e^{-\frac{i\alpha}{r}} F\left(\frac{1}{2}+\beta, 1+2\beta, \frac{2i\alpha}{r}\right), \tag{5}$$

$$R^{(2)} = \left(\frac{1}{r}\right)^{\frac{1}{2}-\beta} e^{-\frac{i\alpha}{r}} F\left(\frac{1}{2}-\beta, 1-2\beta, \frac{2i\alpha}{r}\right). \tag{6}$$

$\alpha = l\sqrt{\omega^2l^2 - m^2}$, $\beta = \sqrt{\mu_0^2l^2 + 1}$ and $F(a,c,z)$ is the confluent hypergeometric function (Kummer's solution). $F(a,c,z) = 1 + \sum_{n=1}^{\infty}\frac{(\alpha)_n}{n!(\gamma)_n}z^n$ and $(\alpha)_0 = 1$; $(\alpha)_n = \alpha(\alpha+1)\cdots(\alpha+n-1)$.

The massless BTZ hole has the same boundary condition as that of the generic BTZ hole that at infinity there is no outgoing wave due to the infinite effective potential. If $\omega = \pm\frac{m}{l}$, then $\alpha = 0$, we obtain

$$R^{(1)} = \left(\frac{1}{r}\right)^{\frac{1}{2}+\beta}, \tag{7}$$

$$R^{(2)} = \left(\frac{1}{r}\right)^{\frac{1}{2}-\beta}. \tag{8}$$

The boundary condition requires B in (4) must be zero. For general case, the constant B also requires to be zero in the spacial infinity and the asymptotic limit of the wave function becomes

$$\Psi_\infty = \frac{1}{\sqrt{r}}A\left(\frac{1}{r}\right)^{\frac{1}{2}+\beta} e^{-\frac{i\alpha}{r}}e^{-i\omega t}e^{im\phi} = Ae^{-i\omega t}e^{im\phi}e^{-\frac{i\alpha}{r}}\frac{1}{r^{1+\beta}}. \tag{9}$$

The required boundary condition is automatically satisfied, which shows that contrary to the generic black hole case where QNMs are proportional to the quantized imaginary part of the frequency, the QNMs in the massless BTZ hole is absent. There is only normal modes in the perturbation for the massless BTZ hole.