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Calculation of massive 2-loop operator matrix elements with outer gluon lines

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Abstract

Massive on-shell operator matrix elements and self-energy diagrams with outer gluon lines are calculated analytically at $O(\alpha_s^2)$, using Mellin–Barnes integrals and representations through generalized hypergeometric functions. This method allows for a direct evaluation without decomposing the integrals using the integration-by-parts method.

1. Introduction

In the asymptotic region $Q^2 \gg m^2$, the heavy flavor contributions to the deeply inelastic structure functions can be obtained from the corresponding massive operator matrix elements and the light flavor Wilson coefficients [1]. The massless Wilson coefficients for deeply inelastic scattering are known up to 3-loop order [2,3]. The heavy flavor contributions were calculated to next-to-leading order in [4] semi-analytically. A fast numerical implementation was given in [5]. Complete analytic results were derived only for the limit $Q^2 \gg m^2$ for the structure function $F_2^{Q\bar{Q}}(x, Q^2)$ to $O(\alpha_s^2)$ [1] and $F_L^{Q\bar{Q}}(x, Q^2)$ to $O(\alpha_s^3)$ [6]. In both cases, the $O(\alpha_s^2)$ massive operator matrix elements are required. The asymptotic contributions cover all logarithmic and the constant terms, while contributions of $O((m^2/Q^2)^k), k \geqslant 1$, are not contained. In the case of the structure function $F_2(x, Q^2)$, these terms yield a very good description already in the region $Q^2 \gtrsim 20\ \text{GeV}^2$, while for $F_L(x, Q^2)$ this approximation only holds at large scales $Q^2 \gtrsim 1000\ \text{GeV}^2$. Since the heavy flavor contributions to the structure functions amount to 20–40% in the small x region, cf. [7], and the scaling violations of these terms differ from that of the light parton contributions, their knowledge is essential for precision measurements of the QCD scale Λ_{QCD} in singlet analyses.

In this Letter we address a new compact calculation of the genuine 2-loop scalar integrals contributing to the massive operator matrix elements with outer gluon lines, based on the Mellin–Barnes technique [8–10] and using representations through generalized hypergeometric functions [11]. This approach allows to thoroughly avoid the use of the integration-by-parts method [12], which keeps the contributing number of terms low and yields very compact results. Moreover, we work in Mellin space to use the appropriate symmetry of the problem leading to further compactification. The complete calculation of the asymptotic heavy flavor Wilson coefficients will be presented elsewhere [13]. In the following, we will outline the principal method and present then the results for the seven contributing two-loop integrals in terms of nested harmonic sums [14,15]. Some of the special sums needed are listed in Appendix A.

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2. The method

The massive 2-loop diagrams considered are shown in Fig. 1. The diagrams contain either three or four massive lines. The $\otimes$-symbol in Fig. 1 denotes the operator insertion of the corresponding local quark–gluon operators, see Fig. 2.

The diagrams can be decomposed into a $*$-product as described in Fig. 3. We follow the calculation of Ref. [10], now generalized from massless self-energy diagrams to massive operator matrix elements.

Also in this case the above decomposition of diagrams can be achieved by applying the Mellin–Barnes representation[1]

$$\frac{1}{(A_1+A_2)^\nu} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\sigma\, A_1^\sigma A_2^{-\nu-\sigma}\,\frac{\Gamma(-\sigma)\Gamma(\nu+\sigma)}{\Gamma(\nu)}. \tag{1}$$

Let us consider diagram I as an example. The corresponding gluing-product is depicted in Fig. 4. Applying the Feynman-parameterization to the 2-point function yields

$$I^{(1,2)} = \frac{\Gamma(\nu_{14})}{\Gamma(\nu_1)\Gamma(\nu_4)}\left(m^2\right)^{\nu_{14}-D/2}(4\pi)^2\int_0^1 dx_1\,dx_2\,x_1^{\nu_1-1}x_2^{\nu_4-1}\delta(x_1+x_2-1)$$
$$\times\int\frac{d^Dk_1}{(2\pi)^D}\frac{(\Delta\,.\,k_1)^{N-1}}{(x_1k_1^2+x_1m^2+x_2(k_1-p)^2+x_2m^2)^{\nu_{14}}}. \tag{2}$$

Here, Δ denotes a light-like vector with $\Delta^2=0$, and $D=4-2\varepsilon$. ν_i is the integer power of the respective propagator and $\nu_{ij\dots}=\nu_i+\nu_j+\cdots$. The calculation is performed in the $\overline{\text{MS}}$ scheme and we factor out $S_\varepsilon=\exp[\varepsilon(\ln(4\pi)-\gamma_E)]$ for each loop.[2] One shifts $k_1\to k_1+x_2p$ and the numerator term is decomposed as

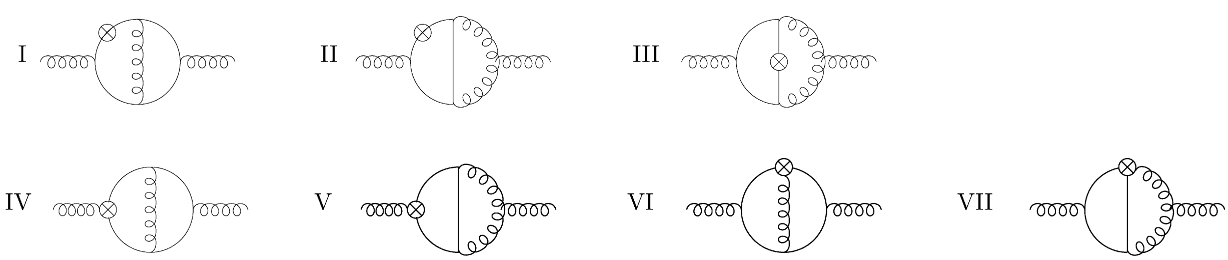
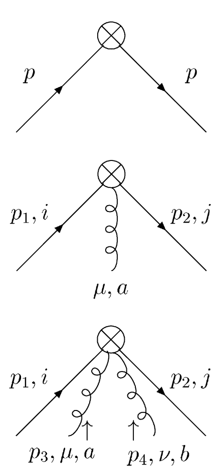


Fig. 1. Genuine 2-loop diagrams contributing to the massive operator matrix elements.



$\not\Delta(\Delta\,.\,p)^{N-1}, \quad N\geqslant 1,$

$g t^a_{ji}\Delta^\mu\not\Delta\sum_{j=0}^{N-2}(\Delta\,.\,p_1)^j(\Delta\,.\,p_2)^{N-j-2}, \quad N\geqslant 2,$

$g^2\Delta^\mu\Delta^\nu\not\Delta\sum_{0\leqslant j<l}^{N-2}\left[(\Delta p_1)^{N-l-2}(\Delta p_1+\Delta p_4)^{l-j-1}(\Delta p_2)^j\left(t^at^b\right)_{ji}\right.$
$\left.+(\Delta p_1)^{N-l-2}(\Delta p_1+\Delta p_3)^{l-j-1}(\Delta p_2)^j\left(t^bt^a\right)_{ji},\right], \quad N\geqslant 3.$

Fig. 2. QCD Feynman rules for the composite local operator insertions.

[1] In his original contribution, Barnes notes that the contour integral representations (1) and those for more complicated integrands date back to Pincherle [17], Mellin [9] and Riemann [18], cf. also [19].

[2] All integrals are normalized to contain no mass-scale or factors of 2π in the final result.