Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 647 (2007) 200–206
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Hawking radiation from rotating black holes in anti-de Sitter spaces via gauge and gravitational anomalies

Qing-Quan Jiang [a,*], Shuang-Qing Wu [b]

[a] *Institute of Particle Physics, Central China Normal University, Wuhan, Hubei 430079, People's Republic of China*
[b] *College of Physical Science and Technology, Central China Normal University, Wuhan, Hubei 430079, People's Republic of China*

Received 23 November 2006; received in revised form 5 January 2007; accepted 19 January 2007

Available online 12 February 2007

Editor: M. Cvetič

Abstract

Robinson–Wilczek's recent work, which treats Hawking radiation as a compensating flux to cancel gravitational anomaly at the horizon of a Schwarzschild-type black hole, is extended to study Hawking radiation of rotating black holes in anti-de Sitter spaces, especially that in dragging coordinate system, via gauge and gravitational anomalies. The results show that in order to restore gauge invariance and general coordinate covariance at the quantum level in the effective field theory, the charge and energy flux by requiring to cancel gauge and gravitational anomalies at the horizon, must have a form equivalent to that of a $(1+1)$-dimensional blackbody radiation at Hawking temperature with an appropriate chemical potential.

PACS: 04.70.Dy; 04.62.+v; 03.65.Sq

Keywords: Hawking radiation; Anomaly; Rotating AdS black holes

1. Introduction

The study of Hawking radiation continued to attract a great deal of attention in theoretical physics since it was first proposed in 1974 [1]. There exist several derivations of Hawking radiation in the literature (see, for example, [2–5]). Among them, one method to derive Hawking radiation is to calculate the energy momentum tensor flux in the black hole background. The Christensen–Fulling's seminal work [5] showed that the trace anomaly, an anomaly in conformal symmetry, can derive important constraints on the energy momentum tensor of quantum fields in a black hole background. Due to the anomaly in conformal symmetry, the flux of Hawking radiation can be treated as that of the trace of the energy momentum tensor. However, their observation was based upon several assumptions: firstly, the background was limited to $(1+1)$ dimensions; secondly, the fields were massless; and finally, there was no back-scattering effect for the massless particles in $(1+1)$ dimensions. Therefore, Hawking radiation appears as a rather special phenomenon in this approach. Recently, Robinson and Wilczek [6] proposed another novel approach which ties Hawking radiation to the cancellation of gravitational anomaly at the horizon of the Schwarzschild-type black hole. Specifically, the scalar field theory in an arbitrary dimensional black hole spacetimes can be reduced to that in a $(1+1)$-dimensional spacetime by using a dimensional reduction technique. In the effective two-dimensional reduction, each partial wave of the scalar field exhibits a general coordinate symmetry. However, when omitting the classically irrelevant ingoing modes at the horizon, the effective theory becomes chiral, and the anomaly with respect to general coordinate symmetry arises to impose great constraints on the energy momentum tensor.[1] Meanwhile, they pointed out that if demanding general

* Corresponding author.
E-mail addresses: jiangqingqua@126.com (Q.-Q. Jiang), sqwu@phy.ccnu.edu.cn (S.-Q. Wu).

[1] In fact, the effective field theory is only basing on the observable physics and defined outside the horizon of the black hole. An observer who lives outside

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.01.058

coordinate covariance at the quantum level to hold in the effective field theory, gravitational anomaly can be cancelled by the quantum effect of the classically irrelevant ingoing mode, and the flux of energy momentum tensor is equal to that of $(1+1)$-dimensional blackbody radiation at Hawking temperature.

In the case of the charged or rotating black holes in arbitrary dimensions, the quantum field near the black hole horizon can be interpreted, by a dimensional reduction technique, as that in the backgrounds of the dilaton (whose contribution to Hawking flux is dropped due to the static background), $(1+1)$-dimensional metric, and the gauge field. However, the effective gauge potential corresponding to the gauge field in the two-dimensional reduction for the charged black holes is originated from the electric field of the black holes, while for rotating black holes it is due to the $U(1)$ gauge symmetry which is associated with the axisymmetry of the black hole. After neglecting the classically irrelevant ingoing modes near the horizon, the effective two-dimensional theory becomes chiral there, and contains gauge and gravitational anomalies to give constraints on the gauge current and the energy momentum tensor. Hawking flux from these two types of black holes can be determined in terms of the values of anomalies at the horizon by demanding gauge invariance and general coordinate covariance at the quantum level [7–10]. Obviously, the derivation of Hawking radiation in this way can be more universal since it is only dependent on the conditions of anomaly cancellation at the horizon, no matter what the black holes in arbitrary dimensions are rotating or charged.

On the other hand, properties of black holes in anti-de Sitter (AdS) spaces especially those of thermodynamics [11] have been investigated thoroughly in recent years within the context of the AdS/CFT correspondence [12]. Thus it is interesting to investigate the Hawking radiation of rotating black holes in AdS spaces from the viewpoint of anomaly cancellation. In this Letter, we shall study Hawking radiation from rotating black holes in AdS spaces via gauge and gravitational anomalies at the horizon. By using a dimensional reduction technique, we can reduce the scalar field near the horizon of the black holes to that in the backgrounds of the dilaton, $(1+1)$-dimensional metric, and the gauge field. Here the contribution of dilaton background to the flux of Hawking radiation is dropped due to a static background. It should be noted that although the dilaton and the conformal factor of the metric share a common form, the gauge potential in the two-dimensional reduction of a Kerr–AdS black hole is originated from the induced symmetry associated with the isometry along the ϕ-direction, and the charge of the field is given by the azimuthal quantum number m, while that of a Kerr–Newman–AdS black hole is composed of one primary gauge potential originated from the electric field of the black hole and another $U(1)$ gauge potential associated with the axisymmetry of the black hole. Since the matter field in the ergosphere near the horizon must be dragged by the gravitational field of a spinning source because there exists a frame-dragging effect of the coordinate in the rotating spacetime, so it is also physically reasonable to investigate this issue in the dragging coordinate system. When do so, one can observe that the $U(1)$ gauge symmetry with respect to the isometry along this ϕ-direction may not be incorporated in the gauge symmetry of the two-dimensional reduction of the Kerr–Newman–AdS black holes. In all, after omitting the classically irrelevant ingoing modes at the horizon, the effective chiral theory for a Kerr–AdS black hole contains a gravitational anomaly and a $U(1)$ gauge anomaly, and that for a Kerr–Newman–AdS black hole is composed of a gravitational anomaly and two gauge anomalies. However, in the case of a Kerr–Newman–AdS black hole in the dragging coordinate system, the effective two-dimensional chiral theory does not contain the $U(1)$ gauge anomaly associated with the induced symmetry originated from the isometry along the ϕ-direction. An observer rest at the dragging coordinate system, which behaves like a kind of locally non-rotating coordinate system, would not observe this $U(1)$ gauge current flux since he is co-rotating with the rotating black hole. If we restore the Boyer–Lindquist coordinate system from the dragging coordinate system, it is also easy to calculate the flux of the angular momentum [8]. In order to maintain gauge invariance or general coordinate covariance at the quantum level to hold in the effective theory, these anomalies are cancelled by the fluxes of $(1+1)$ blackbody radiation at Hawking temperature with appropriate chemical potentials.

This Letter is outlined as follows. In Section 2, Hawking radiation from a Kerr–AdS black hole is investigated by gauge and gravitational anomalies at the horizon, we also prove that the fluxes of gauge current and energy momentum tensor, which are required to cancel the anomalies at the horizon, are exactly equal to that of a $(1+1)$-dimensional blackbody radiation at Hawking temperature with an appropriate chemical potential for an azimuthal angular momentum m. In Section 3, we consider as an example the case of a Kerr–Newman–AdS black hole the Boyer–Lindquist coordinates and in the dragging coordinates, and prove once again that if demanding gauge invariance and general coordinate covariance at the quantum level to hold in the effective theory, the thermal flux of Hawking radiation is capable of cancelling the anomalies at the horizon. Section 4 ends up with some discussions and conclusions.

2. Hawking radiation from a Kerr–AdS black hole

The metric of a four-dimensional Kerr–AdS black hole can be expressed as [13]

a Schwarzschild-type black hole with finite energy cannot observe the physics beyond the horizon of the black hole since the horizon is a null-hypersurface. As the global Killing vector that describes the symmetry of the spacetime is only time-like outside its horizon, we can define the energy of quantum states in an effective theory that only describes observable physics. However, the energy momentum tensor calculated by this definition is divergent at the horizon due to a pile up of the horizon-skimming modes. Thus, the effective theory can properly describe the observable physics with these modes integrated out. Although such formulated effective field theory no longer has observable divergence, it contains gravitational anomaly that imposes great constraints on the energy momentum tensor.