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The study of leading twist light cone wave function of η_c meson

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Abstract

This Letter is devoted to the study of leading twist light cone wave function of η_c meson. The moments of this wave function have been calculated within three approaches: potential models, nonrelativistic QCD and QCD sum rules. Using the results obtained within these approaches the model for the light cone wave function of leading twist has been proposed. Being scale dependent light cone wave function has very interesting properties at scales $\mu > m_c$: improvement of the accuracy of the model, appearance of relativistic tail and violation of nonrelativistic QCD velocity scaling rules. The last two properties are the properties of real leading twist light cone wave function of η_c meson.

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1. Introduction

Commonly exclusive charmonium production at high energies is studied within nonrelativistic QCD (NRQCD) [1]. In the framework of this approach charmonium is considered as a bound state of quark–antiquark pair moving with small relative velocity $v \ll 1$. Due to the presence of small parameter v the amplitude of charmonium production can be built as an expansion in relative velocity v.

Thus in the framework NRQCD the amplitude of any process is a series in relative velocity v. Usually, in the most of applications of NRQCD, the consideration is restricted by the leading order approximation in relative velocity. However, this approximation has two problems which make it unreliable. The first problem is connected with rather large value of relative velocity for charmonium $v^2 \sim 0.3$, $v \sim 0.5$. For this value of v^2 one can expect large contribution from relativistic corrections in any process. So in any process resummation of relativistic corrections should be done or one should prove that resummation of all terms is not crucial. The second problem is connected with QCD radiative corrections. The point is that due to the presence of large energy scale Q there appear large logarithmic terms $(\alpha_s \log Q/m_c)^n$, $Q \gg m_c$ which can be even more important than relativistic corrections at sufficiently large energy ($Q \sim 10$ GeV). So these terms should also be resummed. In principle, it is possible to resum large logarithms in the NRQCD factorization framework [2,3], however such resummation is done rarely.

The illustration of all mentioned facts is the process of double charmonium production in e^+e^- annihilation at B-factories, where leading order NRQCD predictions [4–6] are approximately by an order of magnitude less than experimental results [7,8]. The calculation of QCD radiative corrections [9] diminished this disagreement but did not remove it. Probably the agreement with the experiments can be achieved if, in addition to QCD radiative corrections, relativistic corrections will be resummed [10].

In addition to NRQCD, hard exclusive processes can be studied in the framework of light cone expansion formalism [11,12] where both problems mentioned above can be solved. Within light cone expansion formalism the amplitude is built as an expansion over inverse powers of characteristic energy of the process. Usually this approach is successfully applied for exclusive production of light mesons [12]. However recently the application of light cone expansion formalism to double

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charmonium production [13–16] allowed one to achieve good agreement with the experiments.

In the framework of light cone formalism the amplitude of some meson production in any hard process can be written as a convolution of the hard part of the process, which can be calculated using perturbative QCD, and process independent light cone wave function (LCWF) of this meson that parameterizes nonperturbative effects. From this one can conclude that charmonium LCWFs are key ingredient of any hard exclusive process with charmonium production. This Letter is devoted to the study of leading twist LCWF of η_c meson.

The Letter is organized as follows. In the next section all definitions needed in our calculation will be given. In Section 3 the moments of LCWF will be calculated in the framework of Buchmuller–Tye and Cornell potential models. Section 6 is devoted to the calculation of the moments within NRQCD. QCD sum rules will be applied to the calculation of the moments in Section 5. Using the results obtained in Sections 3–5 the model for LCWF will be built in Section 6. In Section 7 this model will be compared with some other models proposed in literature. In the last section we summarize the results of our Letter.

2. Definitions

The leading twist light cone wave function (LCWF) of η_c meson can be defined as follows [12]

$$\langle 0|\bar{Q}(z)\gamma_\alpha\gamma_5[z,-z]Q(-z)|\eta_c\rangle_\mu = i f_{\eta_c} p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi(\xi,\mu), \tag{1}$$

where the following designations are used: x_1, x_2 are the parts of momentum of the whole meson carried by quark and antiquark correspondingly, $\xi = x_1 - x_2$, p is a momentum of η_c meson, μ is an energy scale. The factor $[z,-z]$, that makes matrix element (1) gauge invariant, is defined as

$$[z,-z] = P\exp\left[ig\int_{-z}^{z} dx^\mu A_\mu(x)\right]. \tag{2}$$

The LCWF $\phi(\xi,\mu)$ is normalized as

$$\int_{-1}^{1} d\xi\,\phi(\xi,\mu) = 1. \tag{3}$$

With this normalization condition the constant f_{η_c} is defined as

$$\langle 0|\bar{Q}(0)\gamma_\alpha\gamma_5 Q(0)|\eta_c\rangle = i f_{\eta_c} p_\alpha. \tag{4}$$

LCWF $\phi(x,\mu)$ can be expanded [12] in Gegenbauer polynomials $C_n^{3/2}(\xi)$ as follows

$$\phi(\xi,\mu) = \frac{3}{4}\left(1-\xi^2\right)\left[1 + \sum_{n=2,4,\ldots} a_n(\mu) C_n^{3/2}(\xi)\right]. \tag{5}$$

At leading logarithmic accuracy the coefficients a_n are renormalized multiplicatively

$$a_n(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\epsilon_n}{b_0}} a_n(\mu_0), \tag{6}$$

where

$$\epsilon_n = \frac{4}{3}\left(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1}\frac{1}{j}\right),$$
$$b_0 = 11 - \frac{2}{3}n_{\rm fl}. \tag{7}$$

It should be noted here that conformal expansion (5) is a solution of Bethe–Salpeter equation with one gluon exchange kernel [11].

From Eqs. (5)–(7) it is not difficult to see that at infinitely large energy scale $\mu \to \infty$ LCWF $\phi(\xi,\mu)$ tends to the asymptotic form $\phi_{\rm as}(\xi) = 3/4(1-\xi^2)$. But at energy scales accessible at current experiments the LCWF $\phi(\xi,\mu)$ is far from its asymptotic form. The main goal of this Letter is to calculate the LCWF $\phi(\xi,\mu)$ of η_c meson. One way to do this is to calculate the coefficients of conformal expansion (5) a_n. Having these coefficients at some energy scale μ_0 one can find the LCWF $\phi(\xi,\mu)$ at any scale μ. Alternative method to parameterize LCWF is to evaluate its moments at some scale

$$\langle\xi^n\rangle_\mu = \int_{-1}^{1} d\xi\,\xi^n\phi(\xi,\mu). \tag{8}$$

In our Letter this method will be used. It is worth noting that since η_c mesons have positive charge parity the LCWF $\phi(\xi,\mu)$ is ξ-even. Thus all odd moments $\langle\xi^{2k+1}\rangle$ equal zero and one needs to calculate only even moments.

Below we will need the formula that connects moment $\langle\xi^n\rangle$ with the matrix element $\langle 0|\bar{Q}\gamma_\nu\gamma_5(iz^\sigma \overset{\leftrightarrow}{D}_\sigma)^n Q|P(p)\rangle$. To obtain it we expand both sides of Eq. (1)

$$\sum_n \frac{i^n}{n!}\langle 0|\bar{Q}\gamma_\nu\gamma_5\left(iz^\sigma \overset{\leftrightarrow}{D}_\sigma\right)^n Q|\eta_c\rangle = i f_{\eta_c} p_\nu \sum_n \frac{i^n}{n!}(zp)^n\langle\xi^n\rangle, \tag{9}$$

and get

$$\langle 0|\bar{Q}\gamma_\nu\gamma_5\left(iz^\sigma \overset{\leftrightarrow}{D}_\sigma\right)^n Q|\eta_c\rangle = i f_{\eta_c} p_\nu (zp)^n\langle\xi^n\rangle. \tag{10}$$

Here

$$\overset{\leftrightarrow}{D} = \vec{D} - \overset{\leftarrow}{D}, \quad \vec{D} = \overset{\rightarrow}{\partial} - igB^a\left(\lambda^a/2\right). \tag{11}$$

3. The moments in the framework of potential models

In potential models charmonium mesons are considered as a quark–antiquark system bounded by some static potential. These models allow one to understand many properties of charmonium mesons. For instance, the spectrum of charmonium family can be very well reproduced in the framework of potential models [17]. Due to this success one can hope that potential models can be applied to the calculation of charmonium equal time wave functions.