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Buckling analyses of composite laminate skew plates with material nonlinearity

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Abstract

A nonlinear material constitutive model, including a nonlinear in-plane shear formulation and the Tsai–Wu failure criterion, for fiber–composite laminate materials is employed to carry out finite element buckling analyses for composite laminate skew plates under uniaxial compressive loads. The influences of laminate layup, plate skew angle and plate aspect ratio on the buckling resistance of composite laminate skew plates are presented. Comparing with the linearized buckling loads of the skew plates, one can observer that the nonlinear in-plane shear together with the failure criterion have significant influence on the ultimate loads of the composite laminate skew plates with $[\pm\theta]_{10S}$ and $[\pm\theta/90/0]_{5S}$ layups but not the $[\alpha/0]_{10S}$ layup.

Keywords: B. Buckling; C. Finite element analysis; Skew plate

1. Introduction

Due to light weight and high strength, the use of fiber–composite laminate materials has been increased rapidly in recent year. The composite laminate plates in service are commonly subjected to compressive forces that may cause buckling. Hence, structural instability becomes a major concern in safe and reliable designs of the composite plates. In the literature, most stability studies of fiber–composite laminate plates have been focused on the rectangular plates [1–10]. Less attention has been paid to the skew laminate plates [11–15]. It is known that the buckling resistance of rectangular composite laminate plates depends on end conditions [4,8], ply orientations [1,2,4,5,8,9], and geometric variables such as aspect ratio, thickness and cutout [3,4,6–8,10]. For skew composite plates, the plate skew angle, α (Fig. 1), should also be a key factor influencing the buckling resistance of the plates [11–15]. It is known that unidirectional fibrous composites exhibit severe nonlinearity in in-plane shear stress-strain relation [16,17]. As a result, the buckling resistance of composite plates is also influenced by the nonlinear behavior of the materials [18].

In this paper, a material model including the nonlinear in-plane shear and the Tsai–Wu failure criterion [19] is reviewed first. Then, nonlinear buckling analyses for simply supported composite skew plates under uniaxial compressive force N (Fig. 1) are carried out using the ABAQUS finite element program [20]. The plates in analysis have various laminate layups, plate skew angles and plate aspect ratios. Numerical results for the material nonlinear buckling behavior of these composite plates are compared with those using linear material properties. Through this study, the influences of laminate layups, plate skew angles and plate aspect ratios on the buckling resistance of skew composite plates are demonstrated.

2. Nonlinear material model for composite materials

2.1. Constitutive modeling of lamina

For fiber–composite laminate materials (Fig. 2), each lamina can be considered as an orthotropic layer in a plane stress condition. Let us define $\Delta\{\sigma'\}=\Delta\{\sigma_1,\sigma_2,\tau_{12}\}^T$, $\Delta\{\tau'_t\}=\Delta\{\tau_{13},\tau_{23}\}^T$, $\Delta\{\varepsilon'\}=\Delta\{\varepsilon_1,\varepsilon_2,\gamma_{12}\}^T$, $\Delta\{\gamma'_t\}=\Delta\{\gamma_{13},\gamma_{23}\}^T$. Then the incremental stress-strain relations for

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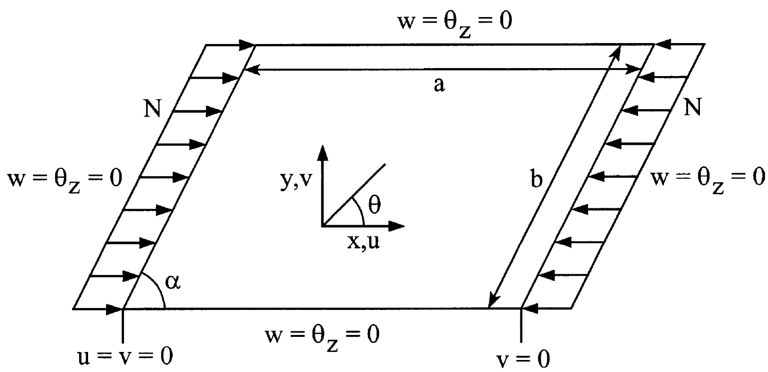

Fig. 1. Composite laminate skew plate with simply supported edge condition.

a linear orthotropic lamina in the material coordinates (1,2,3) can be written as

$$\Delta\{\sigma'\} = [Q_1']\Delta\{\varepsilon'\} \tag{1}$$

$$\Delta\{\tau_t'\} = [Q_2']\Delta\{\gamma_t'\} \tag{2}$$

$$[Q_1'] = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{3}$$

$$[Q_2'] = \begin{bmatrix} \alpha_1 G_{13} & 0 \\ 0 & \alpha_2 G_{23} \end{bmatrix} \tag{4}$$

where α_1 and α_2 are the shear correction factors [21] and are taken to be 0.83 in this study.

It is known that unidirectional fibrous composites exhibit severe nonlinearity in in-plane shear stress-strain relation [16]. Though deviation from linearity is also observed in transverse loading direction, i.e. 2-direction, the degree of this nonlinearity is not comparable to that in the in-plane shear. Therefore, it has been suggested that the nonlinearity associated with the transverse loading direction could be ignored for graphite/epoxy and boron/epoxy [17]. To model the nonlinear in-plane shear behavior, the nonlinear strain-stress relation for a composite lamina suggested by Hahn and Tsai [16] is adopted in this study, which is given as follows:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_{11}} & -\dfrac{\nu_{21}}{E_{22}} & 0 \\ -\dfrac{\nu_{12}}{E_{11}} & \dfrac{1}{E_{22}} & 0 \\ 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + S_{6666}\tau_{12}^2 \begin{Bmatrix} 0 \\ 0 \\ \tau_{12} \end{Bmatrix} \tag{5}$$

In this model only one constant S_{6666} is required to account for the in-plane shear nonlinearity. The value of S_{6666} can be determined by a curve fit to various off-axis tension test data [16]. Inverting and differentiating Eq. (5), we obtain the nonlinear incremental constitutive matrix for the lamina as follows:

$$[Q_1'] = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & \dfrac{1}{1/G_{12}+3S_{6666}\tau_{12}^2} \end{bmatrix} \tag{6}$$

The validity of using Eq. (6) to model the nonlinear in-plane shear has been demonstrated by Hahn and Tsai [16] and is not repeated here. Furthermore, it is assumed that the transverse shear stresses always behave linearly and do not affect the nonlinear behavior of in-plane shear. Hence, the same shear correction factors and shear moduli for transverse shear as those given in Eq. (4) also apply to the cases of nonlinear in-plane shear.

2.2. Failure criterion and degradation of stiffness

Among existing failure criteria, the Tsai–Wu criterion [19] has been extensively used in literature and it is adopted in this analysis. Under plane stress conditions, this failure criterion has the following form:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 = 1 \tag{7}$$

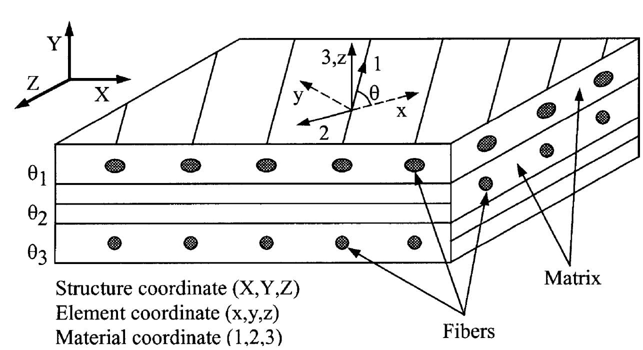

Fig. 2. Material, element and structure coordinates of fiber–composite laminate materials.