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Relic gravitons and viscous cosmologies

Mauricio Cataldo [a,*], Patricio Mella [b]

[a] *Departamento de Física, Facultad de Ciencias, Universidad del Bío–Bío, Avenida Collao 1202, Casilla 5-C, Concepción, Chile*
[b] *Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*

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Abstract

Previously it was shown that there exists a class of viscous cosmological models which violate the dominant energy condition for a limited amount of time after which they are smoothly connected to the ordinary radiation era (which preserves the dominant energy conditions). This violation of the dominant energy condition at an early cosmological epoch may influence the slopes of energy spectra of relic gravitons that might be of experimental relevance. However, the bulk viscosity coefficient of these cosmologies became negative during the ordinary radiation era, and then the entropy of the sources driving the geometry decreases with time. We show that in the presence of viscous sources with a linear barotropic equation of state $p = \gamma\rho$ we get viscous cosmological models with positive bulk viscous stress during all their evolution, and hence the matter entropy increases with the expansion time. In other words, in the framework of viscous cosmologies, there exist isotropic models compatible with the standard second law of thermodynamics which also may influence the slopes of energy spectra of relic gravitons.

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Our universe can be viewed as containing a sea of stochastically distributed gravitational waves of primordial origin. Among all observational cosmological evidences (present and future), primordial gravitational waves should have a sufficiently enlightened character in order to better understand the very early universe. The gravitational waves of cosmological origin are nothing but squeezed states of many gravitons produced from the vacuum fluctuations of the background metric. A qualitative analysis can be performed in the context of different physical frameworks, since all models for the very early universe predict the formation of stochastic gravitational wave backgrounds. As examples we can mention inflationary quintessential models [1], inflationary models in Brans–Dicke theory of gravity [2], cosmological models in the brane-world scenario [3], and superstring theories [4]. The shape of the stochastic graviton background spectrum is affected by the variations of the background dynamics.

In this context, Giovannini [5] has considered the interesting possibility of constructing flat Friedmann–Robertson–Walker (FRW) cosmologies endowed with a bulk viscous stress which induces a violation of the dominant energy condition (DEC) for a limited amount of time at an early cosmological epoch. This kind of cosmological models may be connected to some of the recent remarks of Grishchuk [6] concerning the detectability of stochastic gravitational wave background by forthcoming interferometric detectors, such as LIGO, VIRGO, GEO600, LISA [7]. Effectively, bulk viscous dissipative processes may influence the slopes of the energy spectra of relic gravitons (generated at the time of violation of the DEC) producing an increasing with frequency in a calculable way. These slopes are crucially related to the sign of the $\rho + p$, where ρ and p are, respectively, the energy density and the pressure density of the cosmic fluid. The requirement that one wants expanding and inflationary universes implies that the energy density of the created gravitons cannot increase with frequency if $\rho + p \geqslant 0$, i.e. if the DEC is not violated. Unfortunately, previous models

* Corresponding author.
E-mail addresses: mcataldo@ubiobio.cl (M. Cataldo), patriciomella@udec.cl (P. Mella).

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which exploit this idea have a phase in their evolution where the matter entropy decreases. Specifically was considered a class of solutions which correspond to a viscous fluid with an equation of state given by $p = -\rho$. In this model the early phase (where the DEC is violated) is smoothly connected to a radiation dominated evolution. Depending upon the sign of the bulk viscosity coefficient, the entropy of the sources driving the geometry can very well decrease [8,9].

Let us discuss the class of viscous cosmologies considered in Ref. [5] more in detail. In a flat FRW background the Einstein field equations in the presence of the bulk viscosity coefficient ξ can be written as

$$H^2 = \frac{\kappa}{3}\rho, \tag{1}$$

$$H^2 + \dot{H} = -\frac{\kappa}{6}(\rho + 3P_{\rm eff}), \tag{2}$$

where the effective pressure is given by

$$P_{\rm eff} = p - 3\xi H. \tag{3}$$

In this case $\kappa = 8\pi G$, $H = \dot{a}/a$, $a(t)$ is the scale factor of the flat FRW metric and the overdot represents derivation with respect to the cosmic time coordinate. In order to have the notation of the paper [5] we must identify $M_p^2 = 3/\kappa$ and $p' = P_{\rm eff}$.

Eqs. (1)–(3) imply the energy balance

$$\dot{\rho} + 3H(\rho + P_{\rm eff}) = 0. \tag{4}$$

In order to have a cosmological model whose evolution violates the DEC only for a finite amount of time, in Ref. [5] it is assumed that

$$\kappa\xi = \frac{2}{3}\frac{\dot{H}}{H}. \tag{5}$$

This parametrization is very reasonable since the amount of violation of DEC is proportional to $\dot{H}$. Effectively, from Eqs. (1) and (4) we have that for any solution

$$\rho + P_{\rm eff} = -\frac{2}{\kappa}\dot{H}, \tag{6}$$

and then a violation of the DEC implies that $\dot{H} > 0$.

In order to have a cosmology whose early phase (where the DEC is violated) is smoothly connected to a radiation dominated evolution, Giovannini considers the scale factor given by

$$a(t) = \left(t + \sqrt{t^2 + t_1^2}\right)^{1/2}, \tag{7}$$

and then the self-consistent solution takes the form (see Fig. 1)

$$H = \frac{1}{2\sqrt{t^2 + t_1^2}}, \tag{8}$$

$$\kappa\xi(t) = -\frac{2t}{3(t^2 + t_1^2)}, \qquad \kappa\rho(t) = \frac{3}{4(t^2 + t_1^2)}. \tag{9}$$

One can immediately see by taking the limit $t \to \pm\infty$ that $\xi_{-\infty}(t) > 0$, $\xi_{+\infty}(t) < 0$ and $a_{\pm\infty}(t) \to (\pm t)^{\pm 1/2}$. So $a(t)$ at the final phase of the whole evolution has the behavior of the radiation dominated era.

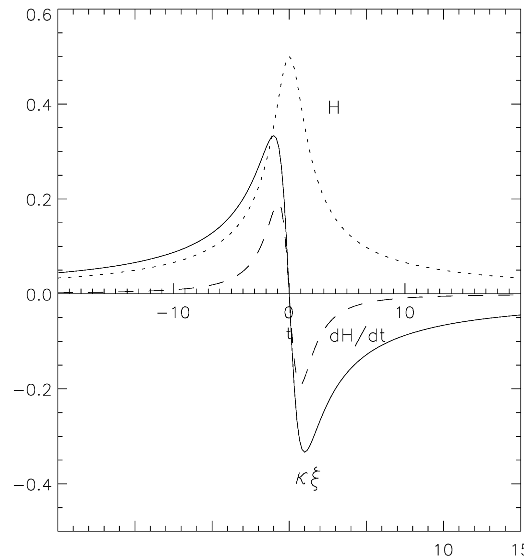

Fig. 1. We show the behavior of H, $\dot{H}$ and $\kappa\xi$ for viscous model (7) and (9). We have chosen $t_1 = 1$. We can see that $\dot{H} > 0$ ($\xi > 0$) for $t < 0$ (DEC is violated) and $\dot{H} < 0$ ($\xi < 0$) for $t > 0$ (DEC is preserved).

Now, if we put the expression (5) into Eq. (6) we conclude that the Giovannini parametrization implies that the local equilibrium pressure is given by $p = -\rho$ [10]. So for $t \to +\infty$ the asymptotic solution to Eqs. (7) and (9) is the following exact solution of the field equations (1)–(4):

$$a(t) = t^{1/2}, \qquad \kappa\xi = -\frac{2}{3t}, \qquad \kappa p = -\kappa\rho = -\frac{3}{4t^2}. \tag{10}$$

Notice that Eqs. (7) and (9) imply that

$$\kappa P_{\rm eff} = \frac{(4t - 3\sqrt{t^2 + t_1^2})}{4(t^2 + t_1^2)^{3/2}}, \tag{11}$$

and then $P_{\rm eff} \to \rho/3$ for $t \to +\infty$, while the local equilibrium pressure behaves always as $p = -\rho$. This is illustrated in Fig. 2.

Let us now consider some physical aspects of the discussed viscous cosmological model. The thermodynamical entropy associated with the bulk viscosity (9) decreases for $t > 0$. This implies that, for large positive cosmic time values, the second law of thermodynamics would be violated [9]. However, as was discussed in Ref. [8], this statement might be not justified since it only takes into account the matter entropy but not the entropy of the geometry itself. This implies that, for a FRW background, in the framework of a well defined extension of the second law of thermodynamics, one should include both the entropy connected with matter and the entropy connected with the FRW background. In this context, for positive cosmic time values, the decrease in the entropy of the sources may be compensated by the growth of the entropy of the FRW background.

Unfortunately, at the present such an extension of the second law of thermodynamics is ambiguous even for cosmological models which do not violate the DEC (see for example [11]). So in this Letter we are interested in finding viscous cosmological models which do not need such an extension of the second law of thermodynamics.