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Target mass corrections to proton spin structure functions and quark–hadron duality

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Abstract

Target mass corrections to proton spin structure functions in deep inelastic scattering region are analysed. Moreover, Bloom–Gilman quark–hadron dualities of proton spin structure functions g_1 and g_2, in inelastic resonance region, are studied. The onsets of the dualities are discussed.

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Keywords: Bloom–Gilman quark–hadron duality; Nucleon spin structure functions; Inelastic resonance region; Target mass corrections

1. Introduction

It is known that the study of quark–hadron duality is essential to understand the physics behind the connection between perturbative QCD (pQCD) and non-perturbative QCD [1], and thus, it shows fundamental issues in strong interaction. In 2001, the new evidence of valence-like quark–hadron duality in nucleon unpolarised structure function F_2 was reported by Jefferson Lab. [2]. Those new data can revisit quark–hadron duality and show that the duality works even in a rather low momentum transfer region of $Q^2 \sim 1\ \mathrm{GeV}^2$. We know that the Bloom–Gilman quark–hadron duality [3] tells that prominent resonances do not disappear relative to background even at a large Q^2. Moreover, the duality also means that the average of the oscillate resonance peaks in the resonance region is the same as that of the scaling structure function at a large Q^2 value. The origin of the Bloom–Gilman quark–hadron duality has been given by Rujula et al. [4] with a QCD explanation. It was also extensively studied in many works [5] through a consideration of the asymptotic perturbative QCD behaviours of the resonance electromagnetic transition amplitudes at a large momentum transfer. Recently, many interesting studies of the quark–hadron duality were published [6–12]. Particularly, Close and Isgur [1] discussed the evolution of the nucleon structure function from coherent resonance region to incoherent inelastic scattering one. Very late review and measurement of the quark–hadron duality are referred to Ref. [13].

So far, one still has no any definitely experimental evidence about the occurrence of the Bloom–Gilman quark–hadron dualities for the nucleon spin structure functions, like g_1 and g_2. It is naturally to expect that the onset of the quark–hadron duality for g_1 of the proton target is at a larger Q^2 point than the Q^2 value of the occurrence of the duality for proton unpolarized structure function F_2 [5]. This is because that very strong Q^2-dependence of g_1 at low Q^2 is needed by the well-known Gerasimov–Drell–Hearn (GDH) sum rule [14]. Experimental study of the quark–hadron duality in the nucleon spin structure function g_1 was performed by HERMES group and Jlab recently [15]. Limited available data indicate that the onset of the duality for g_1 is likely at a larger Q^2 than $1.7\ \mathrm{GeV}^2$. Theoretical analysis also reaches the similar conclusion that the occurrence of the duality of g_1^p is likely at $Q^2 \sim 2\ \mathrm{GeV}^2$ [10].

To study the Bloom–Gilman quark–hadron dualities of the proton spin structure functions g_1 and g_2, one should know the structure functions both in the resonance region with small Q^2 (centre-of-mass energy $W < 2.5$ GeV), and in deep inelas-

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tic scattering (DIS) region. Moreover, the role played by target mass corrections to the structure functions in DIS region should be also carefully analysed. On the one hand, we'll simply regard the parametrization forms of the proton spin structure functions g_1 and g_2 by Simula et al. [16] is a good choice to simulate the spin structure function g_1^p in the resonance region. Those forms contain nucleon elastic effect and the contributions of the resonances and non-resonance background with 14 parameters involved in, and fixed [16] by fitting to the available data in the resonance region. A simple Breit–Wigner shape is used to describe the W dependence of the contribution of an isolated resonance. All four-star resonances, having a total transverse photo-amplitude $\sqrt{|A_{1/2}|^2+|A_{3/2}|^2}$ larger than 0.05 GeV$^{-1/2}$, are considered. On the other hand, we will employ next-to-leading order pQCD calculations for the spin structure functions in the DIS region. There are several known calculations in the literature [17–19], like those by Glück, Reya, Stratmann and Vogelsang (GRSV) [17], and by Leader, Sidorov and Stamenov (LSS) [18]. In this Letter, we shall employ the results of GRSV to analyse the target mass corrections to the spin structure functions in the DIS region. Comparing the averages of the proton spin structure functions both in the resonance and in the DIS regions, we can phenomenologically study the Bloom–Gilman quark–hadron dualities of the proton spin structure functions.

It should be stressed that there were several works related to the target mass corrections for g_1 in the literature [20,21]. The target mass corrections to the Bjorken sum rule were discussed in Ref. [22]. Recently, the target mass corrections to all the nucleon spin structure functions, like g_1 and g_2, have been studied carefully in Refs. [23–26].

This Letter is organized as follows. In Section 2, the target mass corrections to the proton spin structure functions are studied. The corrections to the truncated moments of $g_{1,2}$ and to the quark–hadron dualities of the proton spin structure functions will be discussed in Section 3. The final section is devoted to conclusions.

2. Target mass corrections

One may follow the method proposed by Georgi and Politzer [27], in the case of unpolarised structure function, to get the target mass corrections (TMCs) to the spin structure functions. The recent calculations show that [23,24]

$$g_1^{\mathrm{TMCs}}(x,Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn\, x^{-n}\sum_{j=0}^{\infty}\left(\frac{M^2}{Q^2}\right)^j \frac{n(n+j)!}{j!(n-1)!(n+2j)^2} \times M_1^{n+2j}(Q^2;M=0), \tag{1}$$

where $M_1^n(Q^2;M=0)$ is the Cornwall–Norton (CN) moment of g_1

$$M_1^n(Q^2;M=0) = \int_0^1 dx\, x^{n-1} g_1(x,Q^2;M=0) \tag{2}$$

calculated in the perturbative QCD where all the mass terms $O(M^n/Q^n)$ are neglect, namely, the nucleon mass M vanishes. The explicit twist-2 expression of g_1 with the TMCs is

$$\begin{aligned} g_1^{\mathrm{TMCs}}(x,Q^2) &= \frac{x g_1(\xi,Q^2;M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{4M^2x^2}{Q^2}\frac{x+\xi}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi'}{\xi'} g_1(\xi',Q^2;M=0) \\ &\quad - \frac{4M^2x^2}{Q^2}\frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}} \times \int_\xi^1\frac{d\xi'}{\xi'}\int_{\xi'}^1\frac{d\xi''}{\xi''} g_1(\xi'',Q^2,M=0), \end{aligned} \tag{3}$$

where the Nachtmann variable [20] is

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}} \tag{4}$$

with $x=\frac{Q^2}{Q^2+W^2-M^2}$. Similarly, the spin structure function g_2 with twist-2 contribution and with the TMCs is

$$\begin{aligned} g_2^{\mathrm{TMCs}}(x,Q^2) &= -\frac{x g_1(\xi,Q^2;M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{x(1-4M^2x\xi/Q^2)}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1\frac{d\xi'}{\xi'} g_1(\xi',Q^2;M=0) \\ &\quad + \frac{3}{2}\frac{4M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}} \times \int_\xi^1\frac{d\xi'}{\xi'}\int_{\xi'}^1\frac{d\xi''}{\xi''} g_1(\xi'',Q^2,M=0). \end{aligned} \tag{5}$$

The above g_2^{TMCs} satisfies the well-known Wandzura–Wilczek (WW) relation [28]

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y). \tag{6}$$

Namely, the WW relation is not affected by the target mass corrections.

Here, it should be noted that the Nachtmann moments of the nucleon spin structure functions, shown in the literature [21,22], are

$$\begin{aligned} M_1^n(Q^2;N) &= \int_0^1 dx\frac{\xi^{n+1}}{x^2}\left\{ g_1(x,Q^2)\left[\frac{x}{\xi} - \frac{n^2}{(n+2)^2}\frac{M^2x^2}{Q^2}\frac{\xi}{x}\right] - g_2(x,Q^2)\frac{M^2x^2}{Q^2}\frac{4n}{n+2}\right\}, \quad (n=1,3,5,\ldots) \end{aligned} \tag{7}$$