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V_{us} from hadronic τ decays

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Abstract

We study the reliability of extractions of $|V_{us}|$ based on flavor-breaking hadronic τ decay sum rules. The "(0, 0) spectral weight", proposed previously as a favorable candidate for this extraction, is shown to produce results having poor stability with respect to s_0, the upper limit on the relevant spectral integral, suggesting theoretical errors much larger than previously anticipated. We argue that this instability is due to the poor convergence of the integrated $D = 2$ OPE series. Alternate weight choices designed to bring this convergence under better control are shown to produce significantly improved stability, and determinations of $|V_{us}|$ which are both mutually compatible, and consistent, within errors, with values obtained by other methods.

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1. Background

Three-family unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix implies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \tag{1}$$

with the V_{ub} contribution playing a numerically negligible role [1]. Analyses of $K_{\ell e3}$ incorporating recent updates to the K_L lifetime [2], the K^+ [3], K_L [4] and K_s [5] branching fractions, and the $K_{\ell 3}$ form factor slope parameters [6], together with strong isospin-breaking and long distance electromagnetic corrections computed in the framework of ChPT [7], lead to [8]

$$f_+(0)|V_{us}| = 0.2173 \pm 0.0008, \tag{2}$$

which, with the Leutwyler–Roos estimate, $f_+(0) = 0.961 \pm 0.008$ [9] (compatible within errors with recent quenched and unquenched lattice results [10]), yields [8]

$$|V_{us}| = 0.2261 \pm 0.0021. \tag{3}$$

This result is in good agreement with expectations based on unitarity and the most recent update of the average of superallowed $0^+ \to 0^+$ nuclear β decay [11] and neutron decay [12] results, $|V_{ud}| = 0.9738 \pm 0.0003$ [8]. The $\sim 2\sigma$ discrepancy observed when earlier K decay results were employed thus appears finally to have been resolved. One should, however, bear in mind two recent developments relevant to $|V_{ud}|$: (i) a new measurement of the neutron lifetime, in strong disagreement with the previous world average [13], and (ii) a Penning trap measurement of the Q value of the superallowed ^{46}V decay [14] in significant disagreement with the average used as input in Ref. [11], and with the potential to raise doubts about current evaluations of structure-dependent isospin-breaking corrections [15]. The potentially unsettled $|V_{ud}|$ situation makes alternate (non-$K_{\ell 3}$) determinations of $|V_{us}|$ of interest, both as a means of testing the Standard Model (SM) scenario for strangeness-changing interactions, and for reducing errors through averaging. Two such alternate methods have been proposed recently.

In the first, $|V_{us}/V_{ud}|$ is extracted using lattice results for f_K/f_π in combination with experimental results for $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ [16]. With the recently updated MILC $n_f = 3$ unquenched lattice result, $f_K/f_\pi = 1.198^{+0.016}_{-0.006}$ [17], the first

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method yields

$$|V_{us}| = 0.2245^{+0.0011}_{-0.0031}, \tag{4}$$

compatible within errors with the $K_{\ell 3}$ determination.

The second of these proposals involves the analysis of flavor-breaking sum rules employing strange and non-strange hadronic τ decay data [18], and forms the subject of the rest of this Letter. Existing results, based on the "(0, 0) spectral weight" version of this analysis [18], will be discussed in Section 3.1 below. The discussion to follow represents an update and extension of the preliminary results presented in Ref. [19].

2. V_{us} from hadronic τ decay data

With $\Pi^{(J)}_{V/A;ij}$ the spin J parts of the flavor $ij = ud, us$ vector/axial vector correlators, $\rho^{(J)}_{V/A;ij}$ the corresponding spectral functions, and $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau \text{hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, the kinematics of hadronic τ decay imply [20]

$$R_{V/A;ij} = 12\pi^2 |V_{ij}|^2 S_{\text{EW}} \int_{\text{th}}^{m_\tau^2} \frac{ds}{m_\tau^2} (1 - y_\tau)^2 \times \left[(1 + 2y_\tau) \rho^{(0+1)}_{V/A;ij}(s) - 2y_\tau \rho^{(0)}_{V/A;ij}(s) \right], \tag{5}$$

where $y_\tau = s/m_\tau^2$, V_{ij} is the flavor ij CKM matrix element, $S_{\text{EW}} = 1.0201 \pm 0.0003$ [21] is a short-distance electroweak correction, and the superscript $(0+1)$ denotes the sum of $J = 0$ and $J = 1$ contributions. Eq. (5) is written in such a way that both terms on the RHS can be rewritten using the general finite energy sum rule (FESR) relation,

$$\int_{\text{th}}^{s_0} ds\, w(s) \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds\, w(s) \Pi(s), \tag{6}$$

valid for any analytic weight $w(s)$ and any correlator Π without kinematic singularities. Quantities $R^{(k,m)}_{V/A;ij}$, analogous to $R_{V/A;ij}$, are obtained by rescaling the experimental decay distribution with the factor $(1 - y_\tau)^k y_\tau^m$ before integrating. The corresponding FESR's are referred to as the "(k, m) spectral weight sum rules". Similar FESR's can be written down for general weights $w(s)$, for $s_0 < m_\tau^2$, and for the separate correlator combinations $\Pi^{(0+1)}_{V/A;ij}(s)$ and $s\Pi^{(0)}_{V/A;ij}(s)$. The corresponding spectral integrals, $\int_{\text{th}}^{s_0} ds\, w(s) \rho^{(J)}_{V/A;ij}(s)$, will be denoted $R^w_{ij}(s_0)$ in what follows. In FESR's involving both the $J = 0+1$ and $J = 0$ combinations, the purely $J = 0$ contribution will be referred to as "longitudinal".

With this background, the τ-based extraction of V_{us} works schematically as follows [18]. Given experimental values for the spectral integrals $R^w_{ij}(s_0)$, $ij = ud, us$, corresponding to the same $w(s)$ and same s_0, the combination

$$\delta R^w(s_0) = \frac{R^w_{ud}(s_0)}{|V_{ud}|^2} - \frac{R^w_{us}(s_0)}{|V_{us}|^2} \tag{7}$$

vanishes in the SU(3) flavor limit and hence has an OPE representation, $\delta R^w_{\text{OPE}}(s_0)$, which begins at dimension $D = 2$. Solving for $|V_{us}|$, one has

$$|V_{us}| = \sqrt{\frac{R^w_{us}(s_0)}{[R^w_{ud}(s_0)/|V_{ud}|^2] - \delta R^w_{\text{OPE}}(s_0)}}. \tag{8}$$

At scales ~ 2–$3\ \text{GeV}^2$, and for weights used in the literature, the dominant $D = 2$ term in $\delta R^w_{\text{OPE}}(s_0)$ is much smaller than the leading $D = 0$ contribution and, as a consequence, similarly smaller than the separate ud, us spectral integrals (for physical m_s, typically at the few to several percent level). The OPE uncertainty, $\Delta(\delta R^w_{\text{OPE}}(s_0))$, thus produces a fractional $|V_{us}|$ error $\simeq \Delta(\delta R^w_{\text{OPE}}(s_0))/2R^w_{ud}(s_0)$, *much* smaller than the fractional uncertainty on $\delta R^w_{\text{OPE}}(s_0)$ itself. High accuracy for $|V_{us}|$ is thus obtainable with only modest accuracy for $\delta R^w_{\text{OPE}}(s_0)$ provided experimental spectral integral errors can be kept under control.

At present, the absence of a V/A separation of the us spectral data means one must work with sum rules based on the observed $V + A$ combination. This combination also reduces the fractional ud spectral integral errors. With present ud spectral data [22–24], these errors are at the $\sim 0.5\%$ level for weights used previously in the literature. The much smaller strange branching fraction leads to limited statistics and coarser binning for the us spectral distribution [25–27]. The K pole term is very accurately known, but errors are ~ 6–8% in the K^* region and > 20–30% above 1 GeV^2. For weights used in the literature, the result is us spectral integrals with ~ 3–4% uncertainties [25,27,28]. Experimental errors on $|V_{us}|$ are thus at the ~ 1.5–2% level, and dominated by uncertainties in the us sector. The situation should improve dramatically with the increase in statistics and improved K identification available from the B factory experiments.

A number of points relevant to reducing OPE errors are outlined below. Note that use of the $V + A$ sum rules has the added advantage of strongly suppressing duality violation at the scales considered [29]. Working with weights satisfying $w(s = s_0) = 0$ further suppresses such contributions [29,30], as does working at scales $s_0 > 2\ \text{GeV}^2$ [31].

A major, and irreducible, source of OPE uncertainty for "inclusive" sum rules (those involving both $J = 0 + 1$ and $J = 0$ contributions) is that produced by the bad behavior of the integrated longitudinal $D = 2$ OPE series. This representation displays badly non-convergent behavior, order by order in α_s, even at the maximum scale, $s_0 = m_\tau^2$, allowed by kinematics [32]. Moreover, for the $(k, 0)$ spectral weights, those truncations of this series employed in the literature can be shown to strongly violate constraints associated with the positivity of the continuum (non-K-pole) part of $\rho^{(0)}_{V+A;us}(s)$ [33].

The impossibility of making sensible use of the longitudinal OPE representation necessitates working with sum rules based on the $J = 0 + 1$ combination. Since no complete $J = 0/1$ spin separation of the spectral data exists, a phenomenological subtraction of the longitudinal parts of the experimental decay distribution is necessary. This can be done with good accuracy because the (very accurately known) π and K pole terms dominate the subtraction, for a combination of chiral