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On the quasi-fixed point in the running of CP-violating phases of Majorana neutrinos

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Abstract

Taking the standard parametrization of three-flavor neutrino mixing, we carefully examine the evolution of three CP-violating phases $(\delta, \alpha_1, \alpha_2)$ with energy scales in the realistic limit $\theta_{13} \to 0$. If m_3 vanishes, we find that the one-loop renormalization-group equation (RGE) of δ does not diverge and its running has no quasi-fixed point. When $m_3 \neq 0$ holds, we show that the continuity condition derived by Antusch et al. is always valid, no matter whether the τ-dominance approximation is taken or not. The RGE running of δ undergoes a quasi-fixed point determined by a nontrivial input of α_2 in the limit $m_1 \to 0$. If three neutrino masses are nearly degenerate, it is also possible to arrive at a quasi-fixed point in the RGE evolution of δ from the electroweak scale to the seesaw scale or vice versa. Furthermore, the continuity condition and the quasi-fixed point of CP-violating phases in another useful parametrization are briefly discussed.

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1. Recent solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. The phenomenon of lepton flavor mixing is described by a 3×3 unitary matrix V. A particular parametrization of V has been advocated by the Particle Data Group [5]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 23$ and 13). The phase parameters α_1 and α_2 are commonly referred to as the Majorana CP-violating phases, because they are only physical for Majorana neutrinos and have nothing to do with CP violation in the neutrino–neutrino and antineutrino–antineutrino oscillations. A global analysis of current experimental data yields [6] $30^\circ < \theta_{12} < 38^\circ$, $36^\circ < \theta_{23} < 54^\circ$ and $\theta_{13} < 10^\circ$ at the 99% confidence level. In addition, the neutrino mass-squared differences $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.2\text{–}8.9) \times 10^{-5}\ \text{eV}^2$ and $\Delta m^2_{32} \equiv m^2_3 - m^2_2 = \pm(1.7\text{–}3.3) \times 10^{-3}\ \text{eV}^2$ have been extracted from solar and atmospheric neutrino oscillations at the same confidence level [6]. The sign of Δm^2_{32} remains undetermined and three CP-violating phases of V are entirely unrestricted.

Note that $\theta_{13} = 0$, which may naturally arise from an underlying flavor symmetry (e.g., S_3 [7] or A_4 [8]), is absolutely allowed by the present experimental data. Note also that either $m_1 = 0$ or $m_3 = 0$, which can be obtained from a specific neutrino mass

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model (e.g., the minimal seesaw model [9]), is absolutely consistent with current neutrino oscillation data. These interesting limits deserve some careful consideration in the study of neutrino phenomenology. For instance, the one-loop renormalization-group equation (RGE) of δ includes the $1/\sin\theta_{13}$ term which is very dangerous in the limit $\theta_{13} \to 0$ [10]. It has been noticed by Antusch et al. [11] that the derivative of δ can keep finite when θ_{13} approaches zero, if δ, α_1 and α_2 satisfy a novel continuity condition in the τ-dominance approximation (in which the small contributions of electron and muon Yukawa couplings to the RGEs are safely neglected). It has also been noticed by us [12] that the RGE running of δ may undergo a nontrivial quasi-fixed point driven by the nontrivial inputs of α_1 and α_2 in the tri-bimaximal neutrino mixing scenario [13] with a near mass degeneracy of three neutrinos.

We find it desirable to examine the continuity condition obtained by Antusch et al. [11] without taking the y_τ^2-dominance approximation, where y_τ denotes the tau-lepton Yukawa coupling eigenvalue. The reason is simply that the y_e^2 and y_μ^2 contributions to the RGE of δ may also involve the $1/\sin\theta_{13}$ terms and become dangerous in the limit $\theta_{13} \to 0$. On the other hand, it is desirable to look at possible quasi-fixed points in the RGE running of δ by choosing more generic neutrino mixing scenarios with vanishing (or vanishingly small) θ_{13} and considering different patterns of the neutrino mass spectrum.

The main purpose of this Letter is just to carry out a careful analysis of the RGE evolution of three CP-violating phases $(\delta, \alpha_1, \alpha_2)$ in the realistic limit $\theta_{13} \to 0$ from the electroweak scale $\Lambda_{\rm EW} \sim 10^2$ GeV to the typical seesaw scale $\Lambda_{\rm SS} \sim 10^{14}$ GeV. If m_3 vanishes, we find that the RGE of δ does not diverge and its running has no quasi-fixed point. This new observation demonstrates that our previous understanding of the running behaviors of δ is more or less incomplete. When $m_3 \neq 0$ holds, we show that the continuity condition derived by Antusch et al. can be rediscovered even though the y_e^2 and y_μ^2 contributions to the RGE of δ are not neglected. The RGE running of δ undergoes a quasi-fixed point determined by a nontrivial input of α_2 in the limit $m_1 \to 0$. If three neutrino masses are nearly degenerate (either $\Delta m^2_{32} > 0$ or $\Delta m^2_{32} < 0$), a quasi-fixed point may also show up in the RGE evolution of δ from the electroweak scale to the seesaw scale (or vice versa). Finally we give some brief comments on the continuity condition and the quasi-fixed point of CP-violating phases in another useful parametrization of V.

2. The exact one-loop RGEs of three neutrino masses (m_1, m_2, m_3), three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and three CP-violating phases $(\delta, \alpha_1, \alpha_2)$ have already been derived by Antusch et al. [11] and can be found from the web page [14].[2] Their results, which have been confirmed by Mei and Zhang independently [15], clearly show that only the RGE of δ contains the $1/\sin\theta_{13}$ term. For simplicity, here we only write out the derivative of δ in an exact but compact way:

$$\frac{{\rm d}\delta}{{\rm d}t} = \frac{C(y_\tau^2 - y_\mu^2)}{32\pi^2} \frac{m_3 \chi}{\Delta m^2_{31} \Delta m^2_{32}} \frac{\sin 2\theta_{12} \sin 2\theta_{23}}{\sin\theta_{13}} + \text{other terms}, \tag{2}$$

where $t \equiv \ln(\mu/\Lambda_{\rm SS})$ with μ being an arbitrary renormalization scale below $\Lambda_{\rm SS}$ but above $\Lambda_{\rm EW}$, $C = -3/2$ in the standard model (SM) or $C = 1$ in the minimal supersymmetric standard model (MSSM),

$$\chi = m_3 \Delta m^2_{21} \sin\delta + m_2 \Delta m^2_{31} \sin(\delta + \alpha_2) - m_1 \Delta m^2_{32} \sin(\delta + \alpha_1), \tag{3}$$

and "other terms" stand for those terms which do not include the $1/\sin\theta_{13}$ factor. We find that the y_e^2 contribution to ${\rm d}\delta/{\rm d}t$ does not involve $1/\sin\theta_{13}$ at all, while the $1/\sin\theta_{13}$ terms associated with y_μ^2 and y_τ^2 contributions to ${\rm d}\delta/{\rm d}t$ are identical in magnitude but have the opposite sign. When the τ-dominance approximation is taken (i.e., neglecting the y_e^2 and y_μ^2 contributions in the RGEs), Eq. (2) reproduces the approximate $1/\sin\theta_{13}$ term of ${\rm d}\delta/{\rm d}t$ given in Ref. [11].

In the limit $\theta_{13} \to 0$, which is allowed (and even favored [6]) by current neutrino oscillation data, the $1/\sin\theta_{13}$ term in ${\rm d}\delta/{\rm d}t$ diverges. To keep ${\rm d}\delta/{\rm d}t$ finite, the divergence of $1/\sin\theta_{13}$ has to be canceled by its associate factor. Eq. (2) indicates that $m_3\chi = 0$ needs to be satisfied, in order to cancel the divergence induced by $1/\sin\theta_{13}$ in the limit $\theta_{13} \to 0$. There are two separate possibilities:

(1) $m_3 = 0$. This special but interesting possibility was *not* mentioned in Ref. [11]. In this case, the derivative of δ is apparently finite for vanishing or vanishingly small θ_{13}. Hence the corresponding RGE running of δ is expected to be mild and have no quasi-fixed point. Note that only the difference between α_1 and α_2 has physical significance in the limit $m_3 \to 0$, just like the instructive case in the minimal seesaw model with two heavy right-handed Majorana neutrinos [16]. If both $m_3 = 0$ and $\theta_{13} = 0$ hold at a given energy scale, one can easily show that m_3 and θ_{13} will keep vanishing at any scale between $\Lambda_{\rm EW}$ and $\Lambda_{\rm SS}$ [17].[3] In this particular case, δ is not well defined and has no physical meaning at all energy scales.

(2) $\chi = 0$. In this case, one may arrive at the continuity condition from Eq. (3):

$$\cot\delta = \frac{m_1 \cos\alpha_1 - (1+\zeta) m_2 \cos\alpha_2 - \zeta m_3}{(1+\zeta) m_2 \sin\alpha_2 - m_1 \sin\alpha_1}, \tag{4}$$

where $\zeta \equiv \Delta m^2_{21}/\Delta m^2_{32} \approx \pm(2.2\text{–}5.2) \times 10^{-2}$. Although this result is equivalent to the one derived by Antusch et al. in the τ-dominance approximation [11], it is now obtained by us in no special assumption or approximation. Given $\alpha_1 = \alpha_2 = 0$, the

[2] Note that the Majorana phases φ_1 and φ_2 defined in Refs. [11,14] are equivalent to α_1 and α_2 defined in Eq. (1): $\varphi_i = -\alpha_i$ (for $i = 1, 2$).

[3] Note that the one-loop RGEs of m_i (for $i = 1, 2, 3$) have the form ${\rm d}m_i/{\rm d}t \propto m_i$ [14]. Hence $m_i = 0$ keeps unchanged if it is initially given at one energy scale.