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The reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ and the pion form factor measurements via the radiative return method ☆

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Abstract

The role of the reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ in the pion form factor measurements via the radiative return method without photon tagging is studied in detail. It was shown, that for the KLOE event selection, it gives up to 1% contribution to the reaction $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ for low invariant masses of the two-pion system.

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1. Introduction

Radiative return method of the hadronic cross section extraction from the measurement of the cross section of the reaction $e^+e^- \to \text{hadrons} + \text{photon(s)}$, proposed already some time ago [1], is currently being used by KLOE [2] and BaBar [3] providing very precise experimental data. Further improvement in accuracy is crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, as the error on the hadronic contributions to a_μ may obscure possible new physics signal, seen as a deviation from the Standard Model (SM) predictions. The same information is essential for the evaluation of the running of the electromagnetic coupling (α_{QED}) from its value at low energy up to M_Z as the present error on the hadronic contributions is too big to fully profit from the data of the future ILC (international linear collider) running in the gigaZ mode. For recent reviews of these subjects look [4–6].

The extraction of the cross section $\sigma(e^+e^- \to \text{hadrons})$ from the measured cross section $\sigma(e^+e^- \to \text{hadrons} + \text{photons})$ relies on the factorization

$$d\sigma\left(e^+e^- \to \text{hadrons} + n\gamma\right) = H\, d\sigma\left(e^+e^- \to \text{hadrons}\right), \quad (1)$$

valid at any order for photons emitted from initial leptons, where the function H contains QED radiative corrections. This function is known analytically, if no cuts are imposed, at next to leading order (NLO) and has to be provided in form of an event generator [7,8] of the reaction $e^+e^- \to \text{hadrons} + \text{photons}$ for a realistic experimental setup.

Let us focus on the most important process, where the 'hadrons' means just $\pi^+\pi^-$ pair. This reaction gives the dominant contribution to the hadronic part of a_μ as well as to its error. In case the photon(s) are not measured and only charged pions are tagged, there exists a number of possible backgrounds. It was pointed out in [9], basing on integrated over the whole phase space analytical formulae and containing contributions from diagrams (a) and (d) in Fig. 1, that the reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ can give sizable contributions to the radiative return process, especially for low invariant masses of two pion system. To examine this contribution for a realistic experimental setup, a Monte Carlo program EKHARA was developed.

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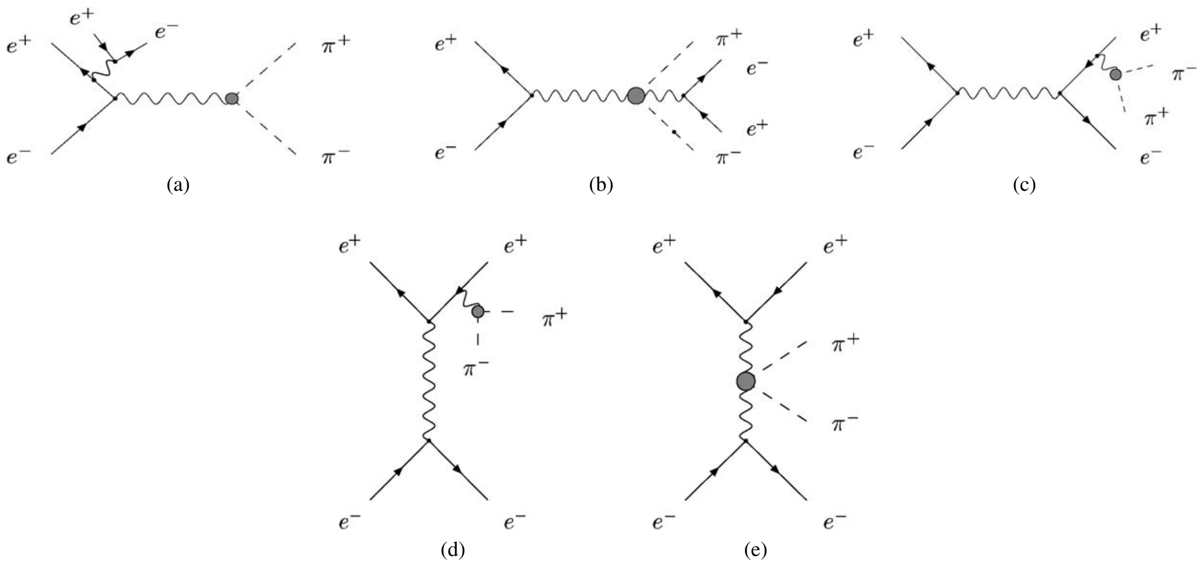

Fig. 1. Diagrams contributing to the process $e^+(p_1)e^-(p_2) \to \pi^+(\pi_1)\pi^-(\pi_2)e^+(q_1)e^-(q_2)$: initial state pair emission (a), final state electron–positron pair emission from $e^+e^- \to \pi^+\pi^-$ diagram (b) final state pion pair emission from s-channel $e^+e^- \to e^+e^-$ diagram (c) pion pair emission from t-channel Bhabha process (d) and $\gamma^*\gamma^*$ pion pair production (e). Only one representative diagram for a given set of diagrams is shown.

Some partial results concerning the electron–positron pair production contributions to the pion form factor measurements and tests of the code were presented in [10,11], while in this letter the amplitude describing the reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ is discussed and results based on the complete tree level amplitude are presented.

2. The scattering amplitude and the generation procedure

As stated already in the introduction, the reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ plays a role in the pion form factor measurement via the radiative return method only if the photons in the reaction $e^+e^- \to \pi^+\pi^- +$ photons are not tagged. This version of the radiative return method was used already by KLOE [2] and as more accurate analysis, based on a significantly bigger data sample, is expected, a detailed study of all possible contributions is obligatory. The complete set of the lowest order diagrams, describing the reaction $e^+e^- \to e^+e^-\pi^+\pi^-$, is shown schematically in Fig. 1.

Helicity amplitude method, with the conventions described in [12,13], was used for the scattering amplitude evaluation. It allows for a fast numerical evaluation and, in addition, all interferences are easily included. Moreover, it partly avoids numerical cancellations present, when one uses the trace method to get the square of the amplitude. To model photon–pion interactions, we use scalar QED (sQED) combined with the vector dominance model (VDM). Within these assumptions, the amplitude has the form

$$M = M_a + M_b + M_c + M_d + M_e, \tag{2}$$

where the amplitudes M_i, $i = a, \dots, e$ correspond to the contributions from the diagrams (a)–(e) from Fig. 1. They read

$$M_a = -\frac{ie^4}{k_1^2 Q^2}\bar{u}(q_2)\gamma_\mu v(q_1)\cdot\bar{v}(p_1) \times\left(\frac{(\gamma^\mu \not{k}_1 - 2p_1^\mu)\not{\Gamma}}{k_1^2 - 2k_1\cdot p_1} + \frac{\not{\Gamma}(2p_2^\mu - \not{k}_1\gamma^\mu)}{k_1^2 - 2k_1\cdot p_2}\right)u(p_2), \tag{3}$$

$$M_b = \frac{-2ie^4 F(s)}{sk_1^2}\bar{v}(p_1)\gamma_\mu u(p_2)\cdot\bar{u}(q_2) \times\left(\gamma^\mu + \frac{2\pi_2^\mu\not{\pi}_1}{k_1^2 + 2\pi_1\cdot k_1} + \frac{2\pi_1^\mu\not{\pi}_2}{k_1^2 + 2\pi_2\cdot k_1}\right)v(q_1), \tag{4}$$

$$M_c = \frac{ie^4}{sQ^2}\bar{u}(q_2)\left(\frac{\gamma^\mu(\not{\Gamma}\not{Q} - 2q_1\cdot\Gamma)}{Q^2 + 2Q\cdot q_1} + \frac{(\not{\Gamma}\not{Q} + 2q_2\cdot\Gamma)\gamma^\mu}{Q^2 + 2Q\cdot q_2}\right)v(q_1)\cdot\bar{v}(p_1)\gamma_\mu u(p_2), \tag{5}$$

$$M_d = \frac{ie^4}{tQ^2}\bar{v}(p_1)\left(\frac{(\not{\Gamma}\not{Q} - 2p_1\cdot\Gamma)\gamma^\mu}{Q^2 - 2Q\cdot p_1} + \frac{\gamma^\mu(\not{\Gamma}\not{Q} - 2q_1\cdot\Gamma)}{Q^2 + 2Q\cdot q_1}\right)v(q_1)\cdot\bar{u}(q_2)\gamma_\mu u(p_2) - \frac{ie^4}{t_1 Q^2}\bar{u}(q_2)\left(\frac{\gamma^\mu(2p_2\cdot\Gamma - \not{Q}\not{\Gamma})}{Q^2 - 2Q\cdot p_2} + \frac{(2q_2\cdot\Gamma + \not{\Gamma}\not{Q})\gamma^\mu}{Q^2 + 2Q\cdot q_2}\right)u(p_2)\bar{v}(p_1)\gamma_\mu v(q_1), \tag{6}$$

$$M_e = \frac{-2ie^4 F(t)F(t_1)}{tt_1}\left(\bar{v}(p_1)\gamma^\mu v(q_1)\bar{u}(q_2)\gamma_\mu u(p_2) + \frac{2\bar{v}(p_1)\not{\pi}_1 v(q_1)\bar{u}(q_2)\not{\pi}_2 u(p_2)}{t_1 - 2\pi_1(p_1 - q_1)} + \frac{2\bar{v}(p_1)\not{\pi}_2 v(q_1)\bar{u}(q_2)\not{\pi}_1 u(p_2)}{t + 2\pi_1(q_2 - p_2)}\right), \tag{7}$$