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New coordinates for BTZ black hole and Hawking radiation via tunnelling

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Abstract

Hawking radiation can usefully be viewed as a semi-classical tunnelling process that originates at the black hole horizon. For the stationary axisymmetric BTZ black hole, a generalized Painlevé coordinate system (Painlevé–BTZ coordinates) is introduced, and Hawking radiation as tunnelling under the effect of self-gravitation is investigated. The corrected radiation is obtained which is not precise thermal spectrum. The result is consistent with the underlying unitary theory. Moreover, Bekenstein–Hawking entropy of BTZ black hole is not necessarily corrected when we choose appropriate coordinate system to study the tunnelling effect.

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1. Introduction

The thermal Hawking radiation [1,2] implies the loss of unitary, and then the breakdown of quantum mechanics [3]. According to Hawking radiation, black hole will radiate its energy away, and vanish in the end. Where does the information go? Although Hawking radiation can be described in a unitary theory in string theory [4], it is not clear how information is returned.

Originally, Hawking explained the existence of black hole radiation as particle's tunnelling coming from vacuum fluctuations near the horizon. The radiation is like electron–positron pair creation in a constant electric field. The energy of a particle can change its sign after crossing the horizon. So a pair created by vacuum fluctuations just inside or outside the horizon can materialize with zero total energy, after one member of the pair has tunnelled to the opposite side. However, Hawking's derivation did not proceed in this way [1]. There were two difficulties to overcome. The first was to find a well-behaved coordinate system at the event horizon. The second was where is the barrier. Recently, a method to describe Hawking radiation as tunnelling process was developed by Kraus and Wilczek [5] and elaborated by Parikh and Wilczek [6–8]. This method gives a leading correction to the emission rate arising from the loss of mass of the black hole corresponding to the energy carried by the radiated quantum. Following this method, the radiation from AdS black hole and de Sitter cosmological horizon were also studied [9–11]. All these spherically symmetric investigations are successful. Because of the complexity of the stationary axisymmetric Kerr black hole [12], the tunnelling effect must be investigated in the dragging coordinate system. The coordinate system and tunnelling result were successfully given in [13]. The picture is: a particle do tunnel out of Kerr black hole, the barrier is created by the outgoing particle itself. If the total energy and angular momentum must be reserved, the outgoing particle must tunnel out a radial barrier to an observer resting in dragging coordinate system.

The black hole solutions of Banados, Teitelboim and Zanelli [14,15] in $(2+1)$ spacetime dimensions are similar as Kerr black hole due to the axisymmetry, so the dragging coordinate system should be used to investigate tunnelling effect naturally. Because the coordinate system is not appropriate in [16], the entropy expression must be modified in order to get

$$\Gamma = e^{-2\,\mathrm{Im}\,I} = e^{+\Delta S}, \tag{1}$$

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where Γ is the emission rate, I is the action for an outgoing positive energy particle, ΔS is the change in the entropy of black hole. In this Letter, we will give a new dragging coordinate system, and show that the complex modified entropy is not necessary in BTZ black hole.

The remainder of the Letter is organized as follows. In Section 2, BTZ black hole metric will be introduced in details. In Section 3, after the dragging coordinate system is given, some features of it will be discussed. In Section 4, we calculate the rate of emission, obtain a correction spectrum without modifying the Bekenstein–Hawking entropy expression. In the end, a brief discussion will be given.

2. BTZ black hole

The BTZ black hole [14,15,17–19] is solution of the standard Einstein–Maxwell equation in $2+1$ spacetime dimensions, with a negative cosmological constant. Using BTZ black hole metric, we can study black hole in a lower-dimension spacetime which could exhibit the key features without the unnecessary complications. Ignoring the Maxwell field, we can write down the action of a three-dimensional theory of gravity as

$$I=\frac{1}{2\pi}\int\sqrt{-g}\left[R+2l^{-2}\right]d^2x\,dt+B, \tag{2}$$

where B is a surface term and the radius l is related to the cosmological constant by $-\Lambda=l^{-2}$. The equations of metric derived from Eq. (2) are solved by the black hole field

$$ds^2=-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)dt^2+\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}dr^2+r^2\left(d\varphi-\frac{J}{2r^2}dt\right)^2, \tag{3}$$

with M the ADM mass, J the angular momentum (spin) of the BTZ black hole and $-\infty<t<+\infty$, $0\leqslant r<+\infty$, $0\leqslant\varphi\leqslant 2\pi$.

For the positive mass black hole spectrum with spin ($J\neq 0$), the line element (3) has two horizons:

$$r_\pm^2=l^2\left\{\frac{M}{2}\left[1\pm\left(1-\left(\frac{J}{Ml}\right)^2\right)^{1/2}\right]\right\}. \tag{4}$$

Of these, r_+, r_- are the black hole outer and inner horizon, respectively. In order for the horizon to exist one must have

$$M>0,\qquad |J|\leqslant Ml.$$

In the extremal case $|J|=Ml$, both roots of $g_{00}=0$ coincide.

The area A_{H} and Hawking temperature T_{H} of the event (outer) horizon are [20,21]

$$A_{\mathrm{H}}=2\pi l\left\{\frac{M}{2}\left[1+\left(1-\left(\frac{J}{Ml}\right)^2\right)^{1/2}\right]\right\}^{1/2}=2\pi r_+, \tag{5}$$

$$T_{\mathrm{H}}=\frac{\sqrt{2}}{2\pi l}\frac{\sqrt{M^2-J^2/l^2}}{(M+\sqrt{M^2-J^2/l^2})^{1/2}}=\frac{1}{2\pi l^2}\left(\frac{r_+^2-r_-^2}{r_+}\right). \tag{6}$$

The entropy of the spinning BTZ black hole is

$$S=4\pi r_+, \tag{7}$$

and if we reinstate the Planck units (since in BTZ units $8\hbar G=1$) we get

$$S=\frac{1}{4\hbar G}A_{\mathrm{H}}=S_{\mathrm{BH}},$$

which is the well-known Bekenstein–Hawking area formula (S_{BH}) for the entropy, and is proven by counting excited states recently in [22].

3. Painlevé–BTZ coordinates

To do a tunnelling computation at the event horizon, we should find a coordinate system that is well-behaved there. At first, we investigate the dragging coordinate system. Let

$$\frac{d\varphi}{dt}=\frac{J}{2r^2}=\Omega, \tag{8}$$

then the line element of BTZ black hole can be rewritten as

$$ds^2=-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)dt^2+\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}dr^2. \tag{9}$$

In fact, the metric Eq. (9) represents a 2-dimensional hypersurface in 3-dimensional BTZ spacetime. This dragging coordinate system is not what we want to use for resolving tunnelling effect in BTZ spacetime. We need another transformation to make none of the components of either the metric or the contra metric diverge at the horizon. Moreover, constant time slices are just flat Euclidean in radial. To obtain a coordinate system analogous to Painlevé coordinates [23], we should perform a coordinate transformation

$$dt=d\tau+f(r)\,dr, \tag{10}$$

where $f(r)$ is a function of r, independent on t.

Putting Eq. (10) into the line element expression (9), we have

$$\begin{aligned}ds^2&=-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)dt^2+\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}dr^2\\&=-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)d\tau^2\\&\quad-2f(r)\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)d\tau\,dr\\&\quad+\left[\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}\right.\\&\quad\left.-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)f^2(r)\right]dr^2.\end{aligned} \tag{11}$$

As a corollary, we demand that the metric is flat Euclidean in radial to the constant-time slices. We then get the condition

$$\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)f^2(r)=1,$$