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Dark energy: the equation of state description versus scalar-tensor or modified gravity

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Abstract

Dark energy dynamics of the universe can be achieved by equivalent mathematical descriptions taking into account generalized fluid equations of state in General Relativity, scalar-tensor theories or modified $F(R)$ gravity in Einstein or Jordan frames. The corresponding technique transforming equation of state description to scalar-tensor or modified gravity is explicitly presented. We show that such equivalent pictures can be discriminated by matching solutions with data capable of selecting the true physical frame.

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1. According to recent astrophysical data, our universe is dominated by a mysterious form of dark energy (DE), for a review and a list of references, see [1], which counts to about 70% of total mass-energy density. As a result, the universe expansion is accelerating. In terms of constant equation of state (EoS) parameterization, observational data indicate that constant EoS parameter is roughly equal to -1. In other words, the accelerating universe could be in the cosmological constant ($w = -1$), quintessence ($-1 < w < -1/3$) or phantom era ($w < -1$). Without taking into account the so-called "first and second coincidence problems" (for a recent discussion, see [2]) the fundamental problem is to select the (correct) EoS for the observed universe consistently related to the early epochs. Even considering (perfect/imperfect) ideal fluid description, there are various possibilities: constant EoS, time-dependent EoS, complicated (explicit or implicit) EoS functional dependence of the pressure from energy density (and time), inhomogeneous EoS, etc. The situation is even more complicated since several proposals for DE (from scalars to string-inspired gravity [3]) exist.

In the present Letter, we develop a technique by which it is possible, whatever the ideal fluid EoS description is, to transform such a fluid in a scalar-tensor theory taking into account the "same" FRW scale factor. However, the process can be reversed. Subsequently, the scalar-tensor theory can be represented as a modified gravity theory (without scalar field) with the same scale factor in Jordan or Einstein frames conformally related. Of course, these three descriptions, leading to the same FRW dynamics, differ in various respects (for instance, the Newton law is different, quantum versions of such theories are not equivalent, nucleosynthesis and LSS can be achieved in different ways, etc.). The proposal is to discriminate among the three approaches to DE considering observational data: in this sense, the "true" selection of mathematically equivalent descriptions is operated at the solution level spanning as much as possible wide ranges of cosmological parameters like the redshift z.

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2. Let us start from the following action:

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R-\frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi-V(\phi)\right\}. \tag{1}$$

Here $\omega(\phi)$ and $V(\phi)$ are functions of the scalar field ϕ. The function $\omega(\phi)$ actually may be chosen to be equal to 1 or -1 as it is shown below. Its possible role is to realize transitions between deceleration/acceleration phases or non-phantom and phantom phases [4]. Let us now assume a spatially-flat FRW metric:

$$ds^2=-dt^2+a(t)^2\sum_{i=1}^{3}\left(dx^i\right)^2 \tag{2}$$

and that the scalar field ϕ only depends on the time coordinate t. Then the FRW equations are given by

$$\frac{3}{\kappa^2}H^2=\rho,\qquad -\frac{2}{\kappa^2}\dot{H}=p+\rho. \tag{3}$$

Here the energy density ρ and the pressure p are

$$\rho=\frac{1}{2}\omega(\phi)\dot{\phi}^2+V(\phi),\qquad p=\frac{1}{2}\omega(\phi)\dot{\phi}^2-V(\phi). \tag{4}$$

Combining (3) and (4), one obtains

$$\omega(\phi)\dot{\phi}^2=-\frac{2}{\kappa^2}\dot{H},\qquad V(\phi)=\frac{1}{\kappa^2}\left(3H^2+\dot{H}\right). \tag{5}$$

The interesting case is that $\omega(\phi)$ and $V(\phi)$ are defined in terms of a single function $f(\phi)$ as

$$\omega(\phi)=-\frac{2}{\kappa^2}f'(\phi),\qquad V(\phi)=\frac{1}{\kappa^2}\left(3f(\phi)^2+f'(\phi)\right). \tag{6}$$

Hence, the following solution are obtained

$$\phi=t,\qquad H=f(t). \tag{7}$$

One can check that the solution (7) satisfies the scalar-field equation:

$$0=\omega(\phi)\ddot{\phi}+\frac{1}{2}\omega'(\phi)\dot{\phi}^2+3H\omega(\phi)\dot{\phi}+V'(\phi). \tag{8}$$

Then *any* cosmology defined by $H=f(t)$ in (7) can be realized by (6).

Since we can always redefine the scalar field ϕ as $\phi\to F(\phi)$ by an arbitrary function $F(\phi)$, we can choose the scalar field to be a time coordinate: $\phi=t$ as in (7). Using (4), one finds

$$\rho=\frac{3}{\kappa^2}f(\phi)^2,$$
$$p=-\frac{3}{\kappa^2}f(\phi)^2-\frac{2}{\kappa^2}f'(\phi). \tag{9}$$

Since $\phi=f^{-1}(\kappa\sqrt{\rho/3})$, we reobtain the equation of the state (EoS):

$$p=-\rho-\frac{2}{\kappa^2}f'\left(f^{-1}\left(\kappa\sqrt{\frac{\rho}{3}}\right)\right), \tag{10}$$

which contains all the cases where the EoS is given by $p=w(\rho)\rho$ (for a recent study on the acceleration of the universe in the EoS descrition, see [5,8] and references therein). Furthermore, since f^{-1} could be always a single-valued function, Eq. (10) contains more general EoS given by

$$0=F(\rho,p). \tag{11}$$

Conversely, if an EoS is given by (11), since ρ and p are given by (9), the corresponding $f(\phi)$ can be obtained by solving the following differential equation:

$$F\left(\frac{3}{\kappa^2}f(\phi)^2,-\frac{3}{\kappa^2}f(\phi)^2-\frac{2}{\kappa^2}\frac{df(\phi)}{d\phi}\right)=0. \tag{12}$$

If we define a new field φ as

$$\varphi=\int d\phi\sqrt{\left|\omega(\phi)\right|}, \tag{13}$$

the action (1) can be rewritten as

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R\mp\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\tilde{V}(\varphi)\right\}. \tag{14}$$

The sign in front of the kinetic term depends on the sign of $\omega(\phi)$. If the sign of ω and then the sign of $\dot{H}$ is positive (negative), the sign of the kinetic term is $-$ $(+)$. Therefore, in the phantom phase, the sign is always $+$ and in the non-phantom phase, always $-$. One assumes ϕ can be solved with respect to φ: $\phi=\phi(\varphi)$. Then the potential $\tilde{V}(\varphi)$ is given by $\tilde{V}(\varphi)\equiv V(\phi(\varphi))$. Since $\tilde{V}(\varphi)$ could be uniquely determined, there is one to one correspondence between H and $\tilde{V}(\varphi)$.

By the variation over φ in the action (14), in the FRW metric, the scalar-field equation follows

$$0=\pm\ddot{\varphi}+3H\dot{\varphi}+\tilde{V}'(\varphi). \tag{15}$$

Since the energy density and the pressure is now given by

$$\rho=\pm\frac{1}{2}\dot{\varphi}^2+\tilde{V}(\varphi),\qquad p=\pm\frac{1}{2}\dot{\varphi}^2-\tilde{V}(\varphi), \tag{16}$$

the conservation of the energy can be obtained by using (15):

$$\begin{aligned}\dot{\rho}+3H(\rho+p)&=\pm\dot{\varphi}\ddot{\varphi}+\tilde{V}'(\varphi)\dot{\varphi}+3H\dot{\varphi}^2\\&=\dot{\varphi}\left(\pm\ddot{\varphi}+3H\dot{\varphi}+\tilde{V}'(\varphi)\right)\\&=0.\end{aligned} \tag{17}$$

Then one can start either from the EoS ideal fluid description or from the scalar-tensor theory (14) description: the emerging cosmology is the same.

3. Let us consider several examples. As first example, the simplest case, we take into account a dust model with $p=0$. Since

$$-\frac{3}{\kappa^2}f(\phi)^2-\frac{2}{\kappa^2}\frac{df(\phi)}{d\phi}=0, \tag{18}$$

one gets

$$f(\phi)=\frac{2}{3\phi}, \tag{19}$$

which gives

$$\varphi=\frac{2}{\kappa\sqrt{3}}\ln\frac{\phi}{\phi_0},$$
$$\tilde{V}(\varphi)=V_0\mathrm{e}^{-\kappa\varphi\sqrt{3}},\qquad V_0\equiv\frac{2}{3\kappa^2\phi_0^2}. \tag{20}$$