Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics Letters B 634 (2006) 195–199
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Signature inversion—manifestation of drift of the rotational axis in triaxial nuclei

Zao-Chun Gao [a], Y.S. Chen [a,b], Yang Sun [c]

[a] *China Institute of Atomic Energy, PO Box 275(18), Beijing 102413, PR China*
[b] *Institute of Theoretical Physics, Academia Sinica, Beijing 100080, PR China*
[c] *Department of Physics and Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, IN 46556, USA*

Received 31 October 2005; received in revised form 8 December 2005; accepted 9 January 2006

Available online 30 January 2006

Editor: W. Haxton

Abstract

A possible scheme of realizing shell model calculations for heavy nuclei is based on a deformed basis and the projection technique. Here we present a new development for odd–odd nuclei, in which one starts with triaxially-deformed multi-quasi-particle configurations, builds the shell-model space through exact three-dimensional angular-momentum-projection, and diagonalizes a two-body Hamiltonian in this space. The model enables us to study the old problem of signature inversion from a different view. With an excellent reproduction of the experimental data in the mass-130 region, the results tend to interpret the phenomenon as a manifestation of dynamical drift of the rotational axis with presence of axial asymmetry in these nuclei.

PACS: 21.60.Cs; 21.10.Re; 23.20.Lv; 27.60.+j

Keywords: Shell model; Angular momentum projection; Triaxial deformation; Signature inversion

There have been many unusual and interesting features discovered in the nuclear high-spin rotational spectra. To describe them, it is not feasible to apply the conventional shell models constructed in a spherical basis. Description of heavy, deformed nuclei has therefore relied mainly on the mean-field approximations [1], or sometimes on the phenomenological particle-rotor model [2]. However, there has been an increasing number of compelling evidences indicating that correlations beyond the mean-field level are important and a proper quantum-mechanical treatment for nuclear states is necessary. It is thus important to develop alternative types of nuclear structure model that can incorporate the missing many-body correlations and make shell-model calculations possible also for heavy nuclei.

This Letter reports on a new theoretical development for doubly-odd systems, in line with the effort of developing shell models using deformed bases [3–6]. The present model employs a triaxially deformed (or γ deformed) basis, constructs the model space by including multi-quasi-particle (qp) states (up to 6-qp), and performs exact three-dimensional angular momentum projection. A two-body Hamiltonian is then diagonalized in this space. The idea has been applied so far only in the simplest case with a triaxially-deformed qp vacuum state [6–8]. The current work is the first attempt to realize the triaxial projected shell model idea in a realistic situation with a configuration mixing in a multi-qp model space. As the first application, we address the long-standing question on signature inversion, a phenomenon which has been widely observed in nuclear rotational spectrum but not been convincingly explained.

This phenomenon has been suggested to relate to one of the intrinsic symmetries in nuclei, which corresponds to "deformation invariance" [9]. Due to this property, rotational energies $E(I)$ (I: total spin of a state) of a high-j band can be split into two branches with $\Delta I = 2$, classified by the signature quantum

E-mail address: ysun@nd.edu (Y. Sun).

0370-2693/$ – see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2006.01.033

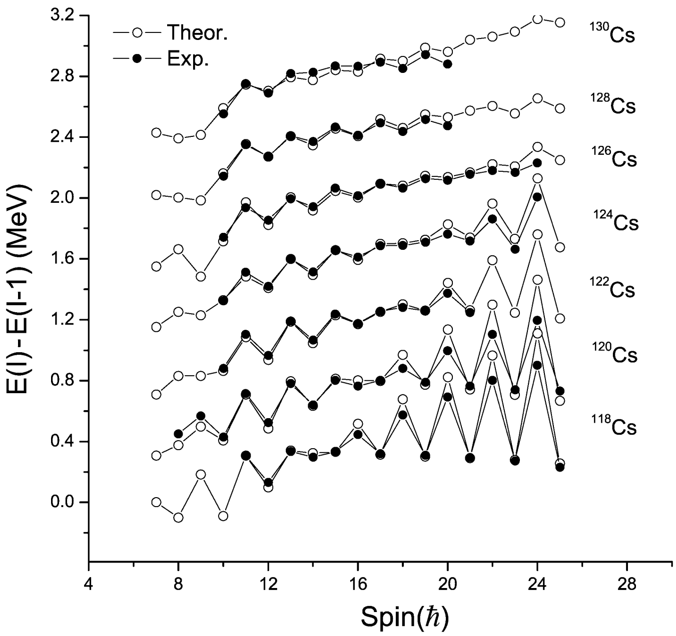

Fig. 1. Comparison of calculated energies with data for the $\pi h_{11/2}\nu h_{11/2}$ bands in $^{118-130}$Cs. Note the increasing trend in the reversion spin: 14.5 for ^{118}Cs, 16.5 for ^{120}Cs, 17.5 for ^{122}Cs, 18.5 for ^{124}Cs, 20.5 for ^{126}Cs, 21.5 for ^{128}Cs (prediction), and 22.5 for ^{130}Cs (prediction). Note also that the bands are shifted vertically by $(A-118)\times 0.2$ MeV where A is mass number.

number, α [2]. As a rule, the energetically favored sequence has $I = j$ (mod 2) and unfavored one $I = j+1$ (mod 2), with j being, for a doubly-odd system, the sum of the angular momenta that the last neutron and the last proton carry. A more general signature rule from a quantum-mechanical derivation was given in Ref. [10]. The critical observation for signature inversion [11] is that at low spins, the energetically unfavored sequence is abnormally shifted downwards, exhibiting, in an $E(I)-E(I-1)$ plot, a reversed zigzag phase to what the signature rule predicts (see Fig. 1). Only beyond a moderate spin I_{rev}, which is called reversion spin [12], is the normal zigzag phase restored. The cause of signature inversion has been a major research subject for many years.

As the first attempt of explanation, triaxiality in the nuclear shape was suggested to be the primary reason [13]. With presence of γ deformation ($0^\circ \leqslant \gamma \leqslant 60^\circ$), the lengths of the two principal axes, the x- and y-axis, are different. If one assumes as usual that the moment of inertia has the same shape dependence as that of irrotational flow, a nucleus prefers to rotate around its intermediate-length principal axis, the y-axis [2]. However, a nucleus with reversed zigzag phase requires a rotation around its shortest principal axis, the x-axis. To describe signature inversion, one had to introduce [14,15] the concept of γ-reversed moment-of-inertia in which one changes the rotation axis by hand. Unsatisfied with this kind of approach, Tajima [16] suggested that other ingredients in addition to triaxiality have to be taken into account. The most popular one discussed in the literature is the neutron–proton interaction [17].

We show that the phenomenon can be naturally described by shell-model-type calculations without invoking unusual assumptions. We first outline our model. The wave-function can be written as

$$|\Psi^{\sigma}_{IM}\rangle = \sum_{K\kappa} f^{\sigma}_{IK_\kappa}\hat{P}^{I}_{MK}|\Phi_\kappa\rangle, \tag{1}$$

in which the projected multi-qp states span the shell model space. In Eq. (1), $|\Phi_\kappa\rangle$ represents a set of 2-, 4-, and 6-qp states associated with the triaxially deformed qp vacuum $|0\rangle$

$$\big\{\alpha^\dagger_{\nu_1}\alpha^\dagger_{\pi_1}|0\rangle, \alpha^\dagger_{\nu_1}\alpha^\dagger_{\nu_2}\alpha^\dagger_{\nu_3}\alpha^\dagger_{\pi_1}|0\rangle, \alpha^\dagger_{\nu_1}\alpha^\dagger_{\pi_1}\alpha^\dagger_{\pi_2}\alpha^\dagger_{\pi_3}|0\rangle, \alpha^\dagger_{\nu_1}\alpha^\dagger_{\nu_2}\alpha^\dagger_{\nu_3}\alpha^\dagger_{\pi_1}\alpha^\dagger_{\pi_2}\alpha^\dagger_{\pi_3}|0\rangle\big\}. \tag{2}$$

The dimension in Eq. (1) is $(2I+1)\times n(\kappa)$, where $n(\kappa)$ is the number of configurations and is typically in the order of 10^2. $\hat{P}^{I}_{MK}$ is the three-dimensional angular-momentum-projection operator [1]

$$\hat{P}^{I}_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega\, D^{I}_{MK}(\Omega)\hat{R}(\Omega), \tag{3}$$

and σ in Eq. (1) specifies the states with the same angular momentum I.

The triaxially deformed qp states are generated by the Nilsson Hamiltonian

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3}\hbar\omega\epsilon_2\left(\cos\gamma\,\hat{Q}_0 - \sin\gamma\,\frac{\hat{Q}_{+2}+\hat{Q}_{-2}}{\sqrt{2}}\right), \tag{4}$$

where the parameters ϵ_2 and γ describe quadrupole deformation and triaxial deformation, respectively. Three major shells ($N = 3, 4, 5$) are considered each for neutrons and protons. Paring correlations are included by a subsequent BCS calculation for the Nilsson states.

The Hamiltonian consists of a set of separable forces

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_\mu \hat{Q}^\dagger_\mu\hat{Q}_\mu - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_\mu \hat{P}^\dagger_\mu\hat{P}_\mu. \tag{5}$$

In Eq. (5), $\hat{H}_0$ is the spherical single-particle Hamiltonian, which contains a proper spin–orbit force [18]. The second term is quadrupole–quadrupole (QQ) interaction that includes the nn, pp, and np components. The QQ interaction strength χ is determined in such a way that it has a self-consistent relation with the quadrupole deformation [3]. The third term in Eq. (5) is monopole pairing, whose strength G_M (in MeV) is of the standard form G/A, with $G = 19.6$ for neutrons and 17.2 for protons, which approximately reproduces the observed odd–even mass differences in this mass region. The last term is quadrupole pairing, with the strength G_Q being proportional to G_M, the proportionality constant being fixed as usual to be 0.16 for all nuclei considered in this Letter. In our model, wherever the quadrupole operator appears, we use the dimensionless quadrupole operator (defined in Section 2.4 of Ref. [3]). We emphasize that no new terms in the Hamiltonian are added and no interaction strengths are individually adjusted in the present work to reproduce data.

To observe a sizable effect of signature, one important condition is that nucleons near the Fermi levels occupy the lower part of high-j shells having smaller K-components. A recent summary for the observed $\pi h_{11/2}\nu h_{11/2}$ bands in the mass-130 region has been given by Hartley et al. [12]. Note that