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Can one extract the π–neutron scattering length from π–deuteron scattering?

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Abstract

We give a proof of evidence that the original power counting by Weinberg can be applied to estimate the contributions of the operators contributing to the π–deuteron scattering length. As a consequence, π–deuteron observables can be used to extract neutron amplitudes—in case of π–deuteron scattering this means that the π–neutron scattering length can be extracted with high accuracy. This result is at variance with recent claims. We discuss the origin of this difference.

1. In absence of neutron targets, it became common practice to use few-body nuclei as effective neutron targets. To extract π–neutron (π–n) amplitudes, π–deuteron (π–d) scattering has been studied in the past. This program can be successful only when both the proton observables and the few-body corrections are known to high accuracy. As the former can be measured directly, they do not cause any problem. For the latter the development of chiral perturbation theory for few-nucleon systems promised a controlled, model-independent, high precision evaluation of the corresponding amplitudes. This program was put forward in a series of publications, e.g., for π–d scattering (see [1] and references therein).

All those analyses are based on the conjecture of Ref. [2] that the transition operators for reactions on nuclei with external sources can be constructed perturbatively within chiral perturbation theory. The resulting operators are then to be convoluted with the appropriate nuclear wave functions. For this to work it needs to be assumed that the contribution of few-nucleon counter terms to the transition operators can be estimated on the basis of naive dimensional analysis. If we apply this recipe to π–d scattering, the leading counter term (Fig. 1(d)) appears at 5th order—two orders down compared to the leading few-body correction (Fig. 1(c)). This was recently confirmed by an explicit calculation of the counter term contribution assuming natural strength for the transition operator [4].

In contrast to this it was found recently that a logarithmic scale dependence shows up in the leading few-body correction to π–d scattering (Fig. 1(c)) that calls for a counter term already at this very order [3,4] (see also [5]), which would preclude any high accuracy extraction of π–n scattering parameters from π–d data. This finding is based on a perturbative treatment of one-pion exchange.

In contrast, in this Letter, we demonstrate by an explicit numerical calculation that the logarithmic divergence disappears, if we treat the one-pion exchange non-perturbatively to obtain the wave function. This explains, why previous studies basically lead to identical numbers for the leading few-body correction although very different wave functions were employed (see discussion in Ref. [1]).[1] Stated differently, we will show that the contact term that necessarily arises at next-to-leading order (NLO), when pions are treated perturbatively in the wave function, can be calculated once the pion exchange is included non-perturbatively in the wave function. This was already conjectured in Ref. [5], but not shown explicitly.

Thus the main goal of our study is to investigate the regulator dependence of the leading few-body correction. Since we are going to employ wave functions that contain non-

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[1] Please note that in Ref. [6], it was shown that in the deuteron channel pions should not be treated perturbatively.

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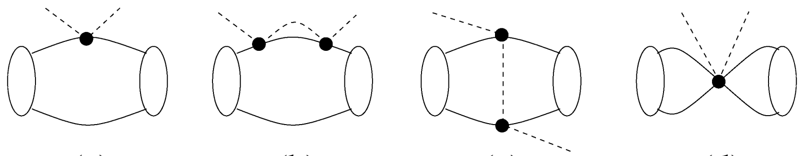

Fig. 1. Typical contributions to π–d scattering. Diagrams (a) and (b) show the tree level and the one-loop contribution to the one-body term, the pion rescattering contribution is depicted by (c) and diagram (d) shows a two-nucleon contact term. In this figure solid (dashed) lines denote nucleons (pions) and ellipses the deuteron wave function.

perturbative pion contributions, this study can only be performed numerically. We will use deuteron wave functions that were constructed for cut-offs that vary over a wide range ($\Lambda = 2$–$20\ \text{fm}^{-1} = 400$–4000 MeV). The procedure of their construction is described in Ref. [8] and will be briefly reviewed below. Already in Ref. [1] a mild cut-off dependence was reported for calculations using wave functions with non-perturbative pions, when the regulator was changed from 500 to 600 MeV. This might either be because of the absence of the logarithmic divergence due to the wave functions used or simply because the coefficient in front of the logarithm is accidentally small. Due to the large range of variation of cut-off values used here we are in the position to answer this question: we will show that there is no sign of a logarithmic regulator dependence of the results as soon as the complete wave functions are used. The consequences of this observation will be discussed in the final section.

In Refs. [9,10], it was stressed that care has to be taken when calculating pion reactions on nuclei. There it was shown that a subtle cancellation pattern exits between contributions from loops in one-body and few-body operators. This has the effect that the static pion exchange is an excellent approximation to the exact result for the leading few-body corrections to π–d scattering. Therefore, here we will focus on the static exchange only.

2. In our investigation we use the wave functions constructed as outlined in Ref. [8]. They emerge as a solution of the Schrödinger equation

$$\Psi^{\pi}_{\Lambda}(p) = G(p)\int d^3p'\,V(p,p')f_{\Lambda}(p,p')\Psi^{\pi}_{\Lambda}(p'), \tag{1}$$

where $G(p) = (-\epsilon - p^2/M)^{-1}$ denotes the two-nucleon propagator with ϵ and M for the deuteron binding energy and the nucleon mass, respectively. The leading order potential $V(p,p')$ comprises contributions from both the one-pion exchange as well as a contact term as depicted in Fig. 2 (see [7]). As regulator function we use

$$f_{\Lambda}(p,p') = \exp\left(\frac{p^4 + p'^4}{\Lambda^4}\right). \tag{2}$$

For a given value of the regulator Λ the only free parameter is C—the strength of the contact term as depicted in Fig. 2(b). For this study, this parameter was adjusted such that the deuteron binding energy was reproduced, to exclude any dependence of the results on an incorrect asymptotic behavior of the deuteron wave function. We checked that the description of the phase shifts in the $^3S_1$–$^3D_1$ channel is comparable to the one obtained in [8]. This numerical study can only be conclusive, when we cover a wide range of cut-offs. We decided to use values of Λ between 2 and 20 fm^{-1} (400–4000 MeV). This range starts below the chiral symmetry breaking scale of $\Lambda_\chi \approx$ 1000–1200 MeV and extends to values larger by a factor of 4. In this range, we also observe the appearance of spurious bound states in the $^3S_1$–$^3D_1$ channel. However, their energies are large

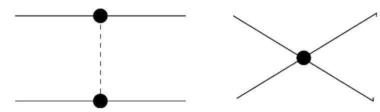

Fig. 2. Contributions to the NN potential at leading order: the one-pion exchange (a) and a momentum-independent contact term (b).

Table 1
Summary of some deuteron properties obtained from Ψ^{π}_{Λ}—the wave functions with non-perturbative one-pion exchange for various cut-offs. Here, the cut-off Λ is given in fm^{-1}, the binding energy and kinetic energy E_0 and T in MeV, the asymptotic S-state normalization A_S is in $\text{fm}^{-1/2}$, the point nucleon radius in fm, and the quadrupole moment in fm^2. η is the ratio of the asymptotic S- and D-state normalization

Λ	E_0	T	P_D	A_S	η	r_d	Q_d
2	2.225	28.91	5.24	0.839	0.030	1.889	0.3005
4	2.225	45.48	8.23	0.866	0.027	1.933	0.2827
6	2.224	62.33	6.94	0.866	0.025	1.932	0.2704
8	2.225	75.95	6.76	0.864	0.026	1.926	0.2676
12	2.227	85.80	7.14	0.864	0.026	1.925	0.2675
16.5	2.214	102.50	7.08	0.862	0.026	1.929	0.2676
20	2.210	115.07	7.07	0.861	0.026	1.929	0.2675
Expt.	2.225	–	–	0.8846	0.0256	1.9671	0.2859

Table 2
Summary of deuteron properties obtained from the wave functions $\Psi^{\text{no}\,\pi}_{\Lambda}$—where only a contact interaction was used in the potential—for various cut-offs. The notation is the same as in Table 1

Λ	E_0	T	A_S	r_d
2	2.225	32.60	0.76	1.728
4	2.225	69.59	0.72	1.617
6	2.225	106.76	0.71	1.585
8	2.225	143.99	0.70	1.569
12	2.225	218.51	0.69	1.555
16	2.225	293.04	0.69	1.547
20	2.225	367.59	0.69	1.543
Expt.	2.225	–	0.8846	1.9671