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Masses of heavy tetraquarks in the relativistic quark model

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Abstract

The masses of heavy tetraquarks with hidden charm and bottom are calculated in the framework of the relativistic quark model. The tetraquark is considered as the bound state of a heavy–light diquark and antidiquark. The light quark in a heavy–light diquark is treated completely relativistically. The internal structure of the diquark is taken into account by calculating the diquark–gluon form factor in terms of the diquark wave functions. New experimental data on charmonium-like states above open charm threshold are discussed. The obtained results indicate that the $X(3872)$ can be the tetraquark state with hidden charm. The masses of ground state tetraquarks with hidden bottom are found to be below the open bottom threshold.

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Recently, significant progress in experimental investigations of charmonium spectroscopy has been achieved. Several new states $X(3872)$, $Z(3931)$, $Y(3943)$, $X(3943)$, $Y(4260)$ were observed [1] which provide challenge to the theory, since not all of them can be easily accommodated as the $c\bar{c}$-charmonium. The most natural conjecture is that these states are the multi-quark composite systems considered long ago e.g. in [2]. The proposal to revisit the multi-quark picture using diquarks has been raised by Jaffe and Wilczek [3]. Currently the best established state is the narrow $X(3872)$ which was originally discovered in B decays [4,5] and later confirmed in $p\bar{p}$ collisions [6,7]. Its mass and observed decays, which favour $J^{PC} = 1^{++}$ assignment, make a $c\bar{c}$ interpretation problematic [8]. Different theoretical interpretations of the $X(3872)$ state were put forward which use the near proximity of its mass to the $D^0\bar{D}^{*0}$ threshold. The most popular ones are: the D^0–$\bar{D}^{*0}$ molecular state bound by pion and quark exchanges [9]; an S-wave cusp at $D^0\bar{D}^{*0}$ threshold [10] and the diquark–antidiquark $[cq][\bar{c}\bar{q}]$ tetraquark state [11] ($q = u, d$).

Maiani et al. [11] in the framework of the phenomenological constituent quark model considered the masses of hidden charm diquark–antidiquark states in terms of the constituent diquark mass and spin–spin interactions. They identified the $X(3872)$ with the S-wave bound state of a spin one and spin zero diquark and antidiquark with the symmetric diquark–spin distribution $([cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1})$ and used its mass to fix the constituent diquark mass. Spin–spin couplings were fixed from the analysis of the observed meson and baryon masses. On this basis they predicted the existence of a 2^{++} state $[cq]_{S=1}[\bar{c}\bar{q}]_{S=1}$ that can be associated to the $Y(3943)$. They also argued [12] that $Y(4260)$ could be the first orbital excitation of the charm-strange diquark–antidiquark state $([cs]_{S=0}[\bar{c}\bar{s}]_{S=0})_{P\text{-wave}}$. In Ref. [13] it is pointed out that non-leptonic B decays provide a favourable environment for the production of hidden charm diquark–antidiquark bound states. In contrast it is argued [14] that the observed $X(3872)$ production in B decays and in high-energy $p\bar{p}$ collisions is too large for a loosely bound molecule (with binding energy of 1 MeV or less).

In this Letter we use the relativistic quark model [15,16] based on the quasipotential approach to calculate the mass spectra of tetraquarks with hidden charm and bottom as the heavy–

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light diquark–antidiquark bound states ($[Qq][\bar{Q}\bar{q}]$, $Q = c, b$). Recently we considered the mass spectra of doubly heavy (QQq) [17] and heavy (qqQ) [18] baryons in the heavy-diquark–light-quark and light-diquark–heavy-quark approximations, respectively. The light quarks and light diquarks were treated completely relativistically. The internal structure of the light and heavy diquarks was taken into account by calculating diquark–gluon form factors on the basis of the determined diquark wave functions. The found good agreement [18] with available experimental data gives additional motivation for considering diquarks as reasonable building blocks of hadrons. It is important to note that all parameters of our model were determined from the previous considerations of meson mass spectra and decays, and we will keep them fixed in the following analysis of heavy tetraquarks.

In the quasipotential approach and diquark–antidiquark picture of heavy tetraquarks the interaction of two quarks in a diquark and the heavy diquark–antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark–quark state and by the tetraquark wave function (Ψ_T) of the bound diquark–antidiquark state respectively, which satisfy the quasipotential equation of the Schrödinger type [15]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}, \tag{3}$$

here $M = E_1 + E_2$ is the bound state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks (q_1 and q_2) which form the diquark or of the diquark (d) and antiquark (d') which form the heavy tetraquark (T), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark–quark or diquark–antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark–antiquark interaction in mesons which were extensively studied in our relativistic quark model [15,16]. For the quark–quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the interaction from the difference in the qq and $q\bar{q}$ colour states.[1]

[1] It is important to study diquark correlations in gauge-invariant color-singlet hadron states on the lattice.

An important role in this construction is played by the Lorentz-structure of the confining interaction. In our analysis of mesons while constructing the quasipotential of the quark–antiquark interaction, we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark–quark and diquark–antidiquark interactions in the tetraquark. The quasipotential is then defined as follows [16,17]:

(a) For the quark–quark (Qq) interaction

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2}\left[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\text{conf}}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V^S_{\text{conf}}(\mathbf{k})\right].$$

Here α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \qquad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right),$$
$$D^{0i} = D^{i0} = 0, \tag{6}$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{7}$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

The effective long-range vector vertex of the quark is defined [16] by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \quad \tilde{k} = (0, \mathbf{k}), \tag{8}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\text{conf}}(r) = (1 - \varepsilon)V_{\text{conf}}(r),$$
$$V^S_{\text{conf}}(r) = \varepsilon V_{\text{conf}}(r), \tag{9}$$

with

$$V_{\text{conf}}(r) = V^S_{\text{conf}}(r) + V^V_{\text{conf}}(r) = Ar + B, \tag{10}$$

where ε is the mixing coefficient.

(b) For diquark–antidiquark ($d\bar{d}'$) interaction

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d E_d}}\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\frac{\langle d'(P')|J_\nu|d'(Q')\rangle}{2\sqrt{E_{d'}E_{d'}}} + \psi^*_d(P)\psi^*_{d'}(P')\left[J_{d;\mu}J^\mu_{d'}V^V_{\text{conf}}(\mathbf{k}) + V^S_{\text{conf}}(\mathbf{k})\right] \times \psi_d(Q)\psi_{d'}(Q'), \tag{11}$$