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Nuclear symmetry energy in relativistic mean field theory

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Abstract

The physical origin of the nuclear symmetry energy is studied within the relativistic mean field (RMF) theory. Based on the nuclear binding energies calculated with and without mean isovector potential for several isobaric chains we confirm earlier Skyrme–Hartree–Fock result that the nuclear symmetry energy strength depends on the mean level spacing $\varepsilon(A)$ and an effective mean isovector potential strength $\kappa(A)$. A detailed analysis of the isospin dependence of these two components contributing to the nuclear symmetry energy reveals a quadratic dependence due to the mean-isoscalar potential, $\sim \varepsilon T^2$, and, completely unexpectedly, the presence of a strong linear component $\sim \kappa T(T+1+\varepsilon/\kappa)$ in the isovector potential. The latter generates a nuclear symmetry energy in RMF theory that is proportional to $E_{\rm sym} \sim T(T+1)$ at variance to the non-relativistic calculation. The origin of the linear term in RMF theory needs to be further explored.

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One of the most important topics in current nuclear physics is to search for the existence limit of atomic nuclei, i.e., to determine the nuclear drip line. In this respect, the role of the continuum in loosely bound nuclei and, in particular, its impact on the treatment of pairing correlations has been discussed to great extent in recent time. However, the proper understanding and correct reproduction of the nuclear symmetry energy (NSE) may have even greater bearing for masses of loosely bound nuclei and certainly is a key issue in the study of exotic nuclei. The very fundamental questions in this respect concern both the understanding of the microscopic origin of the NSE strength as well as its isospin dependence. The latter issue has attracted recently great attention also in $N \sim Z$ nuclei, see Ref. [1] and references therein.

The NSE is conventionally parametrized as:

$$E_{\rm sym} = a_{\rm sym}(A)T(T+\lambda), \tag{1}$$

where $T = |T_z| = |N-Z|/2$. The strength of the NSE admits typically volume and surface components $a_{\rm sym}(A) = a_{\rm v}/A - a_{\rm s}/A^{4/3}$ and its physical origin is traditionally explained in terms of the kinetic energy and mean isovector potential (interaction) contributions, i.e. $a_{\rm sym}(A) = a_{\rm kin}(A) + a_{\rm int}(A)$, respectively [2]. The linear term is found to be strongly model dependent and there is a common belief that mean-field models yield essentially only a quadratic term $\lambda \approx 0$. On the other hand, the nuclear shell-model [3–5] or models restoring isospin symmetry [6] suggest that $\lambda \approx 1$. No consensus is reached so far concerning the value of λ although there is certain preference for $\lambda \approx 1$. Indeed, experimental masses of nuclei with small values of T supports the existence of the linear term [7]. Similar conclusions were reached by Jänecke et al. [8] based on the analysis of experimental binding energies for $A < 80$ nuclei.

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One of the most accurate mass formula, the so-called FRDM [9] employs a value of $\lambda \approx 1$ but inconsistently admits only a volume-like linear term. Assuming $T(T+1)$ dependence Duflo and Zuker have performed a global fit to nuclear masses obtaining [10]

$$a_{\rm sym}(A) = \frac{134.4}{A} - \frac{203.6}{A^{4/3}}\ [\text{MeV}]. \tag{2}$$

A different view on the origin of the NSE was presented recently by Satuła and Wyss. In Refs. [11–13] it was demonstrated using the Skyrme–Hartree–Fock (SHF) model that the NSE can be directly associated with the mean level spacing $\varepsilon(A)$ and mean isovector potential, $E_{\rm sym} = \frac{1}{2}\varepsilon(A)T^2 + \frac{1}{2}\kappa(A)T(T+1)$ [11–13]. Surprisingly, the self-consistent calculations revealed that the complicated isovector mean potential induced by the Skyrme force is similar to that obtained from a simple interaction $\frac{1}{2}\kappa(A)\hat{\mathbf{T}}\cdot\hat{\mathbf{T}}$, i.e., is very accurately characterized by a single strength $\kappa(A)$ [11–13]. This study revealed also that the SHF theory yield in fact a (partial) linear term with $\lambda \approx \kappa/(2a_{\rm sym})$ and that this term originates from neutron–proton exchange interaction.

Alongside with the SHF calculation, the relativistic mean field (RMF) theory has been used for a large variety of nuclear structure phenomena [14]. Since the RMF theory is based on a very different concept from the SHF, it is highly interesting to investigate the structure of the NSE in the framework of the RMF theory.

The details of RMF theory together with its applications can be found in a number of review articles, see for example Ref. [15] and references therein, and will not be repeated here. The basic ansatz of the RMF theory is a Lagrangian density whereby nucleons are described as Dirac particles which interact via the exchange of various mesons [the isoscalar–scalar sigma (σ), isoscalar–vector omega (ω) and isovector–vector rho (ρ)] and the photon. The σ and ω mesons provide the attractive and repulsive part of the nucleon–nucleon force, respectively. The isospin asymmetry is provided by the isovector ρ meson. Hence, by switching on and off the coupling to the ρ meson, one can easily separate the role of isoscalar and isovector parts of the interaction and study them independently.

In the nuclei considered here, time reversal symmetry is preserved and the spatial vector components of $\boldsymbol{\omega}$, $\vec{\boldsymbol{\rho}}$ and $\mathbf{A}$ fields vanish. This leaves only the time-like components ω^0, $\vec{\rho}^{\,0}$ and A^0. Charge conservation guarantees that only the third component of the isovector $\vec{\rho}^{\,0}$ meson is active. For reason of simplicity, axial symmetry is assumed in the present work. The Dirac spinor ψ_i as well as the meson fields can be expanded in terms of the eigenfunctions of a deformed axially symmetric oscillator potential [16] or Woods–Saxon potential [17], and the solution of the problem is transformed into a diagonalization of a Hermitian matrix.

The RMF calculations are performed for the $A = 40$, 48, 56, 88, 100, 120, 140, 160, 164, and 180 isobars with the effective Lagrangians NL3 [18], TM1 [19], and PK1 [20]. Our choice of the parameterizations is somewhat arbitrary. However, the purpose of this work is not to make a detailed comparison to the data but rather to investigate specific features of the RMF approach pertaining to the isovector channel. These properties are expected to be fairly parameterization independent, in particular that these parameterizations reproduce rather well the equation of state for densities $\rho \leqslant 0.2$ fm^{-3} [21,22].

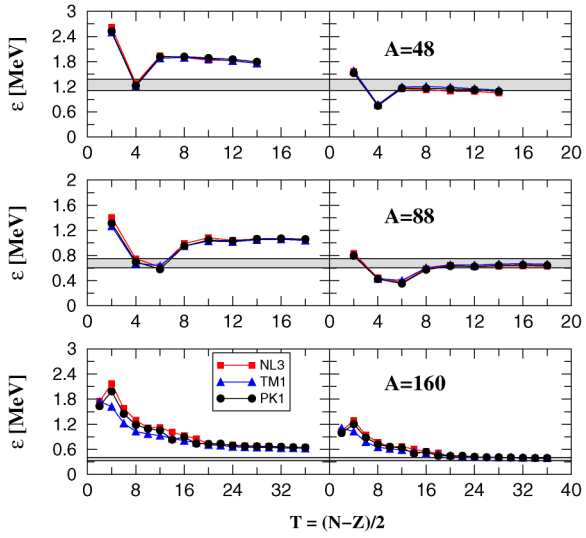

Fig. 1. The mean level spacing ε (left) and its counterpart (right) scaled by m^*/m for $A = 48$ (upper), 88 (middle) and 160 (lower) isobaric chains calculated using effective Lagrangians NL3, TM1, and PK1 as marked in the figure. The shadowed areas correspond to the empirical mean level spacing: $\varepsilon^{\rm emp} = \frac{53}{A} - \frac{66}{A}$ MeV.

The Dirac equations are solved by expansion in the harmonic oscillator basis with 14 oscillator shells for both the fermion fields and boson fields. The oscillator frequency of the harmonic oscillator basis is set to $\hbar\omega_0 = 41A^{-1/3}$ MeV and the deformation of harmonic oscillator basis β_0 is reasonably chosen to obtain the lowest energy. Generally speaking, the RMF calculation reproduce the experimental binding energy to an accuracy less than 1%. For the present study we are mainly interested in the NSE emerging in the RMF theory due to the strong (particle–hole) interaction. Hence the Coulomb potentials and the pairing correlations will be neglected in the following. The full potential in the Dirac equation is

$$\begin{aligned} V_{\rm tot} &= V(\mathbf{r}) + \beta S(\mathbf{r}) \\ &= g_\omega \omega^0(\mathbf{r}) + g_\rho \vec{\tau}\cdot\vec{\rho}^{\,0}(\mathbf{r}) + \beta g_\sigma \sigma(\mathbf{r}). \end{aligned} \tag{3}$$

It can easily be separated into isovector and isoscalar components, i.e., $V_{\rm tot} = V_{\rm is} + V_{\rm iv}$, where

$$\begin{aligned} V_{\rm is}(\mathbf{r}) &= g_\omega \omega^0(\mathbf{r}) + \beta g_\sigma \sigma(\mathbf{r}), \\ V_{\rm iv}(\mathbf{r}) &= g_\rho \vec{\tau}\cdot\vec{\rho}^{\,0}(\mathbf{r}). \end{aligned} \tag{4}$$

The binding energy calculated with the full potential $V_{\rm tot}$ in Eq. (3) is denoted as E_T. The energy obtained by switching off the isovector potential, $V_{\rm iv} \equiv 0$, i.e. by taking in the calculation $V_{\rm tot} \equiv V_{\rm is}$, is denoted by $\tilde{E}_T$. In order to single out the impact of isoscalar fields on the NSE, we use $\tilde{E}_T$ to extract the mean level spacing $\varepsilon(A, T_Z)$ along an isobaric chain

$$\tilde{E}_T(A, T_z) - \tilde{E}_{T=0}(A, T_z = 0) = \frac{1}{2}\varepsilon(A, T_z)T^2. \tag{5}$$