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Electromagnetic corrections to non-leptonic two-body B and D decays

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Abstract

We present analytic expressions to evaluate at $\mathcal{O}(\alpha)$ the effects of soft-photon emission, and the related virtual corrections, in non-leptonic decays of the type $B, D \to P_1 P_2$, where $P_{1,2}$ are scalar or pseudoscalar particles. The phenomenological implications of these results are briefly discussed. For B decays into charged pions the effects of soft-photon emission are quite large: the corrections to the rates can easily exceed the 5% level if tight cuts on the photon energy are applied.

1. Introduction

The large amount of data collected at B factories has allowed to reach statistical accuracies of the order of a few percent on the measurements of several B meson branching fractions. At this level of accuracy electromagnetic effects cannot be neglected. On the one hand, in order to ensure a good control of the experimental efficiencies, it is necessary to include in Monte Carlo simulations the unavoidable emission of soft photons that accompanies all processes with charged particles. On the other hand, the effective cuts applied on the (soft) photon spectra are a key information for a meaningful comparison between theory and experiments.

The theoretical treatment of the infrared singularities generated within QED is a well-known subject and one of the pillars of quantum field theory. A clear and very general discussion can be found, for instance, in the classical papers [1,2]. The general properties of QED have been exploited in great detail in the case of genuine electroweak processes, or processes which can be fully described within perturbation theory within the Standard Model (SM). In these cases there exist both precise theoretical calculations of the electromagnetic (e.m.) corrections and accurate Monte Carlo programs which include the effects of soft-photon emission, such as PHOTOS [3]. Similar tools have not been developed for most exclusive hadronic processes and, in particular, for B and D decays.

Recently, the issue of electromagnetic corrections have received considerable attention in the context of kaon physics [4–7]. As discussed in these works, and as confirmed by recent experimental analyses [8], a correct simulation of electromagnetic corrections is a key ingredient for a precise determination of V_{us} and other effective couplings of weak interactions.

The purpose of the present work is to present simple analytic formulae for he theoretical evaluation, and the numerical simulation, of the leading radiative corrections in B or D meson decays into two scalar or pseudoscalar particles. Given the universal character of the infrared singularities, we evaluate the effects of soft-photon emission (and the corresponding virtual corrections) within scalar QED and in the approximation of a point-like effective weak vertex. The results thus obtained are valid up to constant $\mathcal{O}(\alpha)$ terms (not enhanced by large logs) related to the matching between this effective theory and the "true" theory where the dynamical aspects of weak interactions are taken into account. Moreover, our calculation does not take into account the possible $\mathcal{O}(E_\gamma)$ terms associated to non-bremsstrahlung amplitudes (hard-photon emission).

2. Photon-inclusive widths

The most convenient infrared-safe observable related to the process $H \to P_1 P_2$ is the photon-inclusive width

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$$\Gamma_{12}^{\rm incl}\big(E^{\max}\big) = \Gamma(H \to P_1 P_2 + n\gamma)\big|_{\sum E_\gamma < E^{\max}}, \tag{1}$$

namely the width for the process $H \to P_1 P_2$ accompanied by any number of (undetected) photons, with total missing energy less or equal to $E^{\max}$ in the H meson rest frame. At any order in perturbation theory we can decompose $\Gamma_{12}^{\rm incl}$ in terms of two theoretical quantities: the so-called non-radiative width, Γ_{12}^0, and the corresponding energy-dependent e.m. correction factor $G_{12}(E^{\max})$,

$$\Gamma_{12}^{\rm incl}\big(E^{\max}\big) = \Gamma_{12}^0 G_{12}\big(E^{\max}\big). \tag{2}$$

The energy dependence of $G_{12}(E)$ is unambiguous and universal (i.e., independent from the short-distance dynamics which originate the decay) up to terms which vanish in the limit $E \to 0$. On the contrary, the normalization of $G_{12}(E)$ is arbitrary: we can always move part of the finite (energy-independent) electromagnetic corrections from Γ_{12}^0 to $G_{12}(E)$. Only the product in (2) corresponds to an observable quantity.

In the following we report the explicit expressions of $G_{12}(E)$ as obtained by means of a scalar-QED calculation at $\mathcal{O}(\alpha)$. In order to treat separately infrared (IR) and ultraviolet (UV) divergences, we regulate the former by means of a photon mass and the latter by means of dimensional regularization. We then renormalize the point-like weak coupling in the $\overline{\rm MS}$ scheme.

By construction, we define the non-radiative amplitude Γ_{12}^0 as follows

$$\Gamma_{12}^0 = \frac{\beta}{16\pi M_H}\big|\mathcal{A}_{H\to P_1P_2}(\mu)\big|^2, \tag{3}$$

$$\beta^2 = \big[1-(r_1+r_2)^2\big]\big[1-(r_1-r_2)^2\big], \quad r_i = \frac{M_i}{M_H}, \tag{4}$$

namely the tree-level rate expressed in terms of the renormalized weak coupling. With this convention, the function $G_{12}(E)$ can be written as

$$G_{12}(E) = 1 + \frac{\alpha}{\pi}\left[b_{12}\ln\left(\frac{M_H^2}{4E^2}\right) + F_{12} + \frac{1}{2}H_{12} + N_{12}(\mu)\right], \tag{5}$$

where, following the notation of Ref. [4], we have denoted by H_{12} the finite term arising from virtual corrections, and by F_{12} the energy-independent term generated by the real emission:

$$\int_{E_\gamma<E} \frac{d^3\vec{k}}{(2\pi)^3 2E_\gamma}\sum_{\rm spins}\left|\frac{\mathcal{A}(H\to P_1P_2\gamma)}{\mathcal{A}(H\to P_1P_2)}\right|^2 = \frac{\alpha}{\pi}\left[b_{12}\ln\left(\frac{m_\gamma^2}{4E^2}\right) + F_{12} + \mathcal{O}\left(\frac{E}{M_H}\right)\right]. \tag{6}$$

As expected, after summing real and virtual corrections the infrared logarithmic divergence cancel out in $G_{12}(E)$, giving rise to the universal $\ln(M_H/E)$ terms proportional to

$$b_{+-} = \frac{1}{2} - \frac{4-\Delta_1^2-\Delta_2^2+2\beta^2}{8\beta}\ln\left(\frac{\Delta_1+\beta}{\Delta_1-\beta}\right) + (1\to 2),$$
$$b_{\pm 0} = 1 - \frac{\Delta_1}{2\beta}\ln\left(\frac{\Delta_1+\beta}{\Delta_1-\beta}\right), \tag{7}$$

where $\Delta_{1(2)} = 1 + r_{1(2)}^2 - r_{2(1)}^2$.

Note that $G_{12}(E)$ does depend explicitly on the ultraviolet renormalization scale μ. The scale dependence contained in $N_{12}(\mu)$ cancels out only in the product $\Gamma_{12}^0 G_{12}(E)$ due to the corresponding scale dependence of the weak coupling. As we shall discuss in more detail later on, in practice we are not able to exploit the numerical consequences of this cancellation due to the absence of a first-principle calculation of $\mathcal{A}_{H\to P_1P_2}(\mu)$.

In the $H^0 \to P_1^+ P_2^-$ case we find the following explicit expressions for the coefficients in Eq. (5):

$$\begin{aligned} F_{+-} = {} & \frac{\Delta_1}{2\beta}\ln\left(\frac{\Delta_1+\beta}{\Delta_1-\beta}\right) \\ & + \frac{4-\Delta_1^2-\Delta_2^2+2\beta^2}{4\beta}\left[f\left(-\frac{\beta}{\Delta_1}\right) - f\left(\frac{\beta}{\Delta_1}\right)\right. \\ & - \frac{1}{2}f\left(\frac{\Delta_1-\beta}{2\Delta_1}\right) + \frac{1}{2}f\left(\frac{\Delta_1+\beta}{2\Delta_1}\right) \\ & + \frac{1}{2}\ln(2)\ln\left(\frac{\Delta_1-\beta}{\Delta_1+\beta}\right) + \frac{1}{4}\ln^2\left(1+\frac{\beta}{\Delta_1}\right) \\ & \left. - \frac{1}{4}\ln^2\left(1-\frac{\beta}{\Delta_1}\right)\right] + (1\to 2), \end{aligned} \tag{8}$$

$$\begin{aligned} H_{+-} = {} & \frac{4-\Delta_1^2-\Delta_2^2+2\beta^2}{8\beta} \\ & \times\left[\pi^2 + 2f\left(\frac{\Delta_1+\beta}{2\beta}\right) - 2f\left(-\frac{\Delta_1-\beta}{2\beta}\right)\right. \\ & + \ln^2(\Delta_1-\beta) - \ln^2(\Delta_1+\beta) \\ & \left. + 2\ln\left(\frac{\Delta_1-\beta}{\Delta_1+\beta}\right)\ln\left(\frac{\beta}{2}\right)\right] - 2\ln(2) \\ & + 1 + \frac{1}{2}\beta\ln\left(\frac{\Delta_1+\beta}{\Delta_1-\beta}\right) + \frac{1}{2}(1+\Delta_1)\ln\big({\Delta_1}^2-\beta^2\big) \\ & + (1\to 2), \end{aligned} \tag{9}$$

$$N_{+-} = \frac{3}{4}\ln\left(\frac{\mu^2}{M_H^2}\right) - \frac{3}{4}\ln\big(r_1^2\big) + (1\to 2), \tag{10}$$

where

$$f(x) = {\rm Re}\big[{\rm Li}_2(x)\big] = -\int_0^x \frac{dt}{t}\ln|1-t|. \tag{11}$$

In the $H^\pm \to P_1^\pm P_2^0$ case we find:

$$\begin{aligned} F_{\pm 0} = {} & 1 + \frac{\Delta_1}{2\beta}\ln\left(\frac{\Delta_1+\beta}{\Delta_1-\beta}\right) \\ & - \frac{\Delta_1}{4\beta}\left[\ln^2\left(\frac{\Delta_1-\beta}{\Delta_1+\beta}\right) + 4f\left(\frac{2\Delta_1}{\Delta_1+\beta}\right)\right], \end{aligned} \tag{12}$$

$$\begin{aligned} H_{\pm 0} = {} & -\frac{\Delta_1}{\beta}\left[\frac{1}{2}\ln^2(\Delta_1+\beta) - \frac{1}{2}\ln^2(\Delta_1-\beta) + f\left(\frac{\Delta_2+\beta}{2\beta}\right)\right. \\ & - f\left(-\frac{\Delta_2-\beta}{2\beta}\right) - f\left(1+\frac{(\Delta_2-\beta)(\Delta_1-\beta)}{4\beta}\right) \\ & + f\left(1-\frac{(\Delta_2+\beta)(\Delta_1+\beta)}{4\beta}\right) \\ & - \ln(\Delta_2-\beta)\ln(\Delta_1+\beta) + \ln(\Delta_2+\beta)\ln(\Delta_1-\beta) \end{aligned}$$