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Unified phantom cosmology: Inflation, dark energy and dark matter under the same standard

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Abstract

Phantom cosmology allows to account for dynamics and matter content of the universe tracing back the evolution to the inflationary epoch, considering the transition to the non-phantom standard cosmology (radiation/matter dominated eras) and recovering the today observed dark energy epoch. We develop the unified phantom cosmology where the same scalar plays the role of early time (phantom) inflaton and late-time dark energy. The recent transition from decelerating to accelerating phase is described too by the same scalar field. The (dark) matter may be embedded in this scheme, giving the natural solution of the coincidence problem. It is explained how the proposed unified phantom cosmology can be fitted against the observations which opens the way to define all the important parameters of the model.

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1. According to recent astrophysical data the (constant) effective equation of state (EOS) parameter w_{eff} of dark energy lies in the interval: $-1.48 < w_{\mathrm{eff}} < -0.72$ [1] (see very recent comparison of observational data from different sources in [2], and see also [3]). It is clear that standard Λ-CDM cosmology is in full agreement with observations. Nevertheless, it remains the possibility that universe is currently in its phantom DE phase (for recent study of phantom cosmology, see [4–6] and refs therein). Despite the fact that it remains unclear how decelerating FRW world transformed to the accelerating DE universe, one can try to unify the early time (phantom?) inflation with late time acceleration [4]. In fact, the phantom inflation has been proposed in [7]. The unified inflation/acceleration universe occurs in some versions of modified gravity [8] as well as for complicated EOS of the universe [9] (for recent discussion of similar (phantom) EOS, see [10] and time-dependent viscosious EOS [11]).

In the present Letter we consider unified phantom cosmology with the account of dark matter. Due to the presence of scalar dependent function in front of kinetic term, the same scalar field may correspond to the (phantom) inflaton at very early universe, quintessence at the intermediate epoch and DE phantom at the late universe. The recent transition from decelerating phase to the accelerating phase is naturally described there too. On the same time it is shown that both phantom phases are stable against small perturbations, and that coincidence problem may be naturally solved in our unified model. The equivalent description of the same unified phenomena via the (multi-valued) EOS is given too. In the final section we explain how the proposed unified phantom cosmology can be fitted against the observations which gives the way to define all the important parameters of the model.

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2. Let us start from the following action:

$$S = \int d^4x \sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\} + S_m. \tag{1}$$

Here $\omega(\phi)$ and $V(\phi)$ are functions of the scalar field ϕ and S_m is the action for matter field. Without matter, such an action has been proposed in [12] for the unification of early-time inflation and late-time acceleration in frames of phantom cosmology. We now assume the spatially-flat FRW metric $ds^2 = -dt^2 + a(t)^2\sum_{i=1}^{3}(dx^i)^2$. Let the scalar field ϕ only depends on the time coordinate t. Then the FRW equations are given by

$$\frac{3}{\kappa^2}H^2 = \rho + \rho_m, \qquad -\frac{2}{\kappa^2}\dot{H} = p + \rho + p_m + \rho_m. \tag{2}$$

Here ρ_m and p_m are the energy density and the pressure of the matter respectively. The energy density ρ and the pressure p for the scalar field ϕ are given by

$$\rho = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi), \qquad p = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi). \tag{3}$$

Combining (3) and (2), one finds

$$\omega(\phi)\dot{\phi}^2 = -\frac{2}{\kappa^2}\dot{H} - (\rho_m + p_m),$$
$$V(\phi) = \frac{1}{\kappa^2}\left(3H^2 + \dot{H}\right) - \frac{\rho_m - p_m}{2}. \tag{4}$$

As usually ρ_m and p_m satisfy the conservation of the energy:

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{5}$$

As clear from the first equation (2), in case without matter ($\rho_m = p_m = 0$), when $\dot{H}$ is positive, which corresponds to the phantom phase, ω should be negative, that is, the kinetic term of the scalar field has non-canonical sign. On the other hand, when $\dot{H}$ is negative, corresponding to the non-phantom phase, ω should be positive and the sign of the kinetic term of the scalar field is canonical. If we restrict in one of phantom or non-phantom phase, the function $\omega(\phi)$ can be absorbed into the field redefinition given by $\varphi = \int^\phi d\phi\sqrt{\omega(\phi)}$ in non-phantom phase or $\varphi = \int^\phi d\phi\sqrt{-\omega(\phi)}$ in phantom phase. Usually, at least locally, one can solve ϕ as a function of φ, $\phi = \phi(\varphi)$. Then the action (1) can be rewritten as

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R \mp \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tilde{V}(\varphi)\right\} + S_m. \tag{6}$$

Here $\tilde{V}(\varphi) \equiv V(\phi(\varphi))$. In the sign $\mp$ of (6), the minus sign corresponds to the non-phantom phase and the plus one to the phantom phase. Then both of $\omega(\phi)$ and $V(\phi)$ in the action (1) do not correspond to physical degrees of freedom but only one combination given by $\tilde{V}(\varphi)$ has real freedom in each of the phantom or non-phantom phase and defines the real dynamics of the system. The redefinition, however, has a discontinuity between two phases. When explicitly keeping $\omega(\phi)$, the two phases are smoothly connected with each other (kind of phase transitions). Hence, the function $\omega(\phi)$ gives only redundant degree of freedom and does not correspond to the extra degree of freedom of the system (in the phantom or non-phantom phase). It plays the important role just in the point of the transition between the phantom phase and non-phantom phase. By using the redundancy of $\omega(\phi)$, in any physically equivalent model, one may choose, just for example, $\omega(\phi)$ as $\omega(\phi) = \omega_0(\phi - \phi_0)$ with constants ω_0 and ϕ_0. If we further choose ω_0 to be positive, the region given by $\phi > \phi_0$ corresponds to the non-phantom phase, the region $\phi < \phi_0$ to the phantom phase, and the point $\phi = \phi_0$ to the point of the transition between two phases.

First we consider the case that the parameter w_m in the matter EOS is a constant: $w_m = p_m/\rho_m$. (In principle, such dark matter may be presented via the introduction of one more scalar field.) Then by using (5), one gets $\rho_m = \rho_{m0}a^{-3(1+w_m)}$. Here ρ_{m0} is a constant. If $\omega(\phi)$ and $V(\phi)$ are given by a single function $g(\phi)$ as

$$\omega(\phi) = -\frac{2}{\kappa^2}g''(\phi) - \frac{w_m + 1}{2}g_0\mathrm{e}^{-3(1+w_m)g(\phi)},$$
$$V(\phi) = \frac{1}{\kappa^2}\left(3g'(\phi)^2 + g''(\phi)\right) + \frac{w_m - 1}{2}g_0\mathrm{e}^{-3(1+w_m)g(\phi)}, \tag{7}$$

with a positive constant g_0, we find a solution of (2) or (4) given by

$$\phi = t, \qquad H = g'(t) \qquad \left(a = a_0\mathrm{e}^{g(t)},\ a_0 \equiv \left(\frac{\rho_{m0}}{g_0}\right)^{\frac{1}{3(1+w_m)}}\right). \tag{8}$$

Hence, even in the presence of matter, any required cosmology defined by $H = g'(t)$ can be realized by (7).

More generally, one may consider the generalized EOS like [10]: $p_m = -\rho_m + F(\rho_m)$. Here $F(\rho)$ is a proper function of ρ_m. Using the conservation of the energy (5) gives $a = a_0\mathrm{e}^{-\frac{1}{3}\int\frac{d\rho_m}{F(\rho_m)}}$. Let us assume the above equation can be solved with respect to ρ_m as $\rho_m = \rho_m(a)$. Then if we may choose $\omega(\phi)$ and $V(\phi)$ by a single function $g(\phi)$ as

$$\omega(\phi) = -\frac{2}{\kappa^2}g''(\phi) - F\left(\rho_m\left(a_0\mathrm{e}^{g(\phi)}\right)\right),$$
$$V(\phi) = \frac{1}{\kappa^2}\left(3g'(\phi)^2 + g''(\phi)\right) - \rho_m\left(a_0\mathrm{e}^{g(\phi)}\right) + \frac{1}{2}F\left(\rho_m\left(a_0\mathrm{e}^{g(\phi)}\right)\right), \tag{9}$$

with a positive constant a_0, we find a solution of (2) or (4) again:

$$\phi = t, \qquad H = g'(t) \quad \left(a = a_0\mathrm{e}^{g(t)}\right). \tag{10}$$

Hence, any cosmology defined by $H = g'(t)$ can be realized by (9).

Since the second FRW equation is given by

$$p = -\frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right), \tag{11}$$

by combining the first FRW equation, the EOS parameter w_{eff} looks as

$$w_{\mathrm{eff}} = \frac{p}{\rho} = -1 - \frac{2\dot{H}}{3H^2}. \tag{12}$$

Now it is common to believe that about 5 billion years ago, the deceleration of the universe has turned to the acceleration. We now show that the model describing such a transition could