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Remarks on global anomalies in RCFT orientifolds

B. Gato-Rivera [a,b], A.N. Schellekens [a,b,c,*]

[a] NIKHEF Theory Group, Kruislaan 409, 1098 SJ Amsterdam, The Netherlands
[b] Instituto de Matemáticas y Física Fundamental, CSIC, Serrano 123, Madrid 28006, Spain
[c] IMAPP, Radboud Universiteit, Nijmegen, The Netherlands

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Abstract

We check the list of supersymmetric standard model orientifold spectra of Dijkstra, Huiszoon and Schellekens for the presence of global anomalies, using probe branes. Absence of global anomalies is found to impose strong constraints, but in nearly all cases they are automatically satisfied by the solutions to the tadpole cancellation conditions.

1. Introduction

In previous papers [1,2] coauthored by one of us a large number of supersymmetric open string spectra was found with a chiral spectrum that exactly matches the standard model spectrum. These models were constructed using orientifolds of tensor products of $N = 2$ minimal models. The standard model gauge groups arise due to Chan–Paton multiplicities of boundary states of the underlying rational conformal field theory.

In contrast to the majority of published work on orientifold model building (see, e.g., [3] and references therein), the construction of [2] is algebraic and not geometric. It is based on rational conformal field theory (RCFT) on surfaces with boundaries and crosscaps. The basic RCFT building blocks and the way they are put together are subject to a set of constraints which are the result of many years of work by several groups.

The constraints can be divided into world-sheet and space–time conditions. The boundary states themselves must satisfy the "sewing constraints" [4–8]. There are further constraints on the crosscap states needed to define non-orientable surfaces [9, 10]. These are all worldsheet conditions needed to guarantee the correct factorization of all amplitudes. In addition, some space–time conditions must be imposed, the tadpole cancellation conditions. They are needed to make sure that tree-level one-point functions of closed string states on the crosscap cancel those of the disk. If these tadpoles are left uncancelled, this will manifest itself in the form of infinities in sum of the Klein bottle, annulus and Möbius diagrams.

If the tadpoles correspond to physical states in the projected closed string spectrum, these infinities merely signal that the corresponding string theory is unstable and might be stabilized by shifting the vacuum expectation value of the corresponding field. However, if the tadpoles do not correspond to physical states their presence implies a fundamental inconsistency in the theory, which may manifest itself in the form of chiral anomalies in local gauge or gravitational symmetries.

There is no proof that the aforementioned set of conditions is sufficient to guarantee consistency of the resulting unoriented, open string theories. It was shown in [11] that for the simple current boundary states derived in [12] and that where used in [2] all sewing constraints are satisfied in the oriented case. To our knowledge, however, there is still no complete proof in the unoriented case, although important progress was made in [13]. Nevertheless, the boundary and crosscap states used in the construction are based on generic simple current modifications of the Cardy boundary states [14] and the Rome crosscap formula [15]. They have been successfully compared with geometric constructions, for example, the circle and its orbifold

* Corresponding author.
E-mail address: t58@nikhef.nl (A.N. Schellekens).

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[16] and WZW models. In addition, they can be shown to yield integral partition functions in all cases, a highly non-trivial requirement [17].

With regard to space–time consistency there is a more concrete reason to worry. In other constructions of orientifold models it was observed that in certain cases gauge groups with global anomalies can occur, even though all tadpole conditions are satisfied [18–20]. By "global anomalies" we mean here anomalies in the global definition of field theory path integral, as first described in [21]. The symptom for such an anomaly is an odd number of massless fermions in the vector representation of a symplectic factor of the gauge group (including in particular doublets of $SU(2)$).

Examples of orientifold spectra having such a problem were found in geometric settings, where the problem can be traced back to uncancelled K-theory charges of branes and O-planes. It is known that D-branes are not characterized by (co)homology but by K-theory [22,23]. Tadpole cancellation guarantees, in particular, the cancellation of cohomology charges of branes, which are characterized by long range RR fields coupling to these charges. This cancellation is physically necessary for branes and O-planes that fill all non-compact dimensions, since the field of an uncancelled charge cannot escape to infinity. However, the branes may carry additional Z_2-charges without a corresponding long range field. Tadpole cancellation does not imply the cancellation of these charges. If they remain uncancelled, this may manifest itself in the form of global anomalies.

This implies that also in algebraic constructions one has to be prepared for the possibility of additional constraints. Unfortunately, a complete description of global anomalies in theories of unoriented open strings does not seem to be available at present. Therefore, the best we can do is to examine if the symptoms of the problem are present.

This check was not done systematically for the results presented in [2]. However, since all gauge groups and representations were stored, we have been able to do an a posteriori check. This leads to the following results. The total number of complete spectra at our disposal is 270 058.[1] Of the 270 058 models, only 1015 turn out to have one or more globally anomalous symplectic factors. Interestingly, on average the anomalous models have more than one anomalous symplectic factor: there are 2075 anomalous symplectic factors out of a total of 845 513.

The gauge groups of these models usually (but not always) have "hidden sectors" in addition to the standard model gauge group $SU(3) \times SU(2) \times U(1)$. Since the standard model itself is free of global anomalies, the origin of the anomaly is always related to the hidden sector, but this may happen in two ways. First of all, a symplectic factor within the hidden sector may be anomalous. However, the hidden sector can also cause an $SU(2)$ factor (the weak gauge group or, in a subclass of models, an additional $Sp(2)$ factor) of the standard model to be anomalous. Since we require any open string stretching between the standard model and hidden branes to be non-chiral, this can only happen if an open string has one end on the standard model $SU(2)$ and the other end on a brane with an $O(N)$ Chan–Paton group, with N odd (if the other end is on a symplectic brane the ground state dimension is automatically even, and when it ends on a complex brane the ground state must be a non-chiral pair, again yielding an even multiplicity). This does indeed occur in a few of the aforementioned anomalous cases.

Even if the massless spectrum does not exhibit this problem, this still does not guarantee that the corresponding string theory is globally consistent. Indeed, if one starts with string theory with a globally anomalous $Sp(2)$ factor, moving the two symplectic branes away from the orientifold plane produces a $U(1)$ theory which presumably is also globally inconsistent, but which does not exhibit the problem in its field theory limit. We assume here and below that continuously moving branes cannot introduce or remove such an inconsistency.

A more powerful constraint was suggested in [18]. In addition to the CP-factors or branes present in a given model, one may introduce "probe-branes". The idea is to add a brane–antibrane pair to a given brane configuration, which we assume to have a field theory limit without global anomalies (and that is tadpole-free, and hence has also no local anomalies). The reason for adding such pair rather than a single brane is that the brane and antibrane cancel each others cohomological brane charges, and hence one does not introduce couplings to long range RR fields. This implies that the result is at least free of local chiral anomalies. If that were not the case, a discussion of global anomalies would not make much sense. The resulting configuration is not free of all tadpoles (dilaton tadpoles will not cancel, for instance), and, in particular, is neither supersymmetric nor stable, but that should not affect the consistency.

The CP gauge group of the new configuration can now be checked for global anomalies. Since the probes are added in pairs, they cannot introduce new global field theory anomalies in the existing configuration, but if the CP groups of the probe-brane pair are symplectic, one may find that the latter gauge groups have a global anomaly (i.e., an odd number of vectors). In that case one should conclude that the original theory was inconsistent as well.

It is not clear that this constraint captures all possible global string anomalies. In the context of our RCFT construction, we have for every choice of $N = 2$ tensor product and modular-invariant partition function a definite number of distinct boundary states at our disposal. For a given orientifold choice, a certain subset of those boundaries will have symplectic CP-factors. Each of them can (and will) be used, with its antibrane, as a probe brane pair. If the algebraic model is viewed from a geometric point of view, perhaps additional branes can be considered that do not have an algebraic description, and that would lead to additional constraints if used as probes. In all cases studied in the literature, the branes not present in the algebraic description simply correspond to rational branes continuously moved to non-rational positions. If this is also true for the more complicated cases considered here, this would not yield anything new. The fact that the set of RCFT boundaries is

[1] This is larger than the number of spectra mentioned in [2] because the latter were obtained after identifying spectra modulo hidden sector details. In other words, some of the 270 058 stored spectra differ only in the hidden sector.