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Relativistic non-instantaneous action-at-a-distance interactions

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Abstract

Relativistic action-at-a-distance theories with interactions that propagate at the speed of light in vacuum are investigated. We consider the most general action depending on the velocities and relative positions of the particles. The Poincaré invariant parameters that label successive events along the world lines can be identified with the proper times of the particles provided that certain conditions are imposed on the interaction terms in the action. Further conditions on the interaction terms arise from the requirement that mass be a scalar. A generic class of theories with interactions that satisfy these conditions is found. The relativistic equations of motion for these theories are presented. We obtain exact circular orbits solutions of the relativistic one-body problem. The exact relativistic one-body Hamiltonian is also derived. The theory has three components: a linearly rising potential, a Coulomb-like interaction and a dynamical component to the Poincaré invariant mass. At the quantum level we obtain the generalized Klein–Gordon–Fock equation and the Dirac equation.

In the past thirty years or so, a great deal of work has been focused on the problem of relativistic bound states [1–14], particularly on the relativistic equations for quark–antiquark bound states and the problem of deriving the meson spectrum [15,16]. Within the relativistic action-at-a-distance formulation of Wheeler and Feynman [17–22] (for electrodynamics) solutions of the two-body Dirac equations [23,24] have been found for positronium [23]. The spectrum obtained by this method agrees with the result of quantum field theory [25,26] at least up to the α^4 order. The approach has also been applied to mesons [27]. There is strong experimental evidence that for large separations the interaction between quarks can be effectively described by a linearly rising potential [16]. Several relativistic generalizations of a linearly rising potential have been studied [28–32]. From quantum chromodynamics, using the Wilson loops approach [33], it has been established that the quark–antiquark bound states are effectively described by a static potential, which is a sum of a linearly rising potential and a Coulomb-like interaction: $V = \sigma r - \frac{k}{r}$ [34].

In this Letter we extend the approach of Wheeler and Feynman to explore what types of interparticle interactions are allowed in special relativity. We assume that the interactions travel at the speed of light in vacuum and that the theory can be described by an action principle for which the interaction terms in the action do not depend on the four-vector accelerations or on higher derivatives.

We find explicitly the most general theory that satisfies these conditions. In the static limit we find the theory has three components: a linearly rising potential, a Coulomb-like interaction and a dynamical component to the Poincaré invariant mass.

We obtain the relativistic equations of motion for N particles and apply these results to the relativistic one-body problem, for which we obtain explicitly the Hamiltonian. Quantum mechanical equations for spinless particles and for spin-$\frac{1}{2}$ particles are presented at the end of the Letter. The possibility of considering the effect of a dynamical component to the quark masses in current phenomenological models is naturally suggested by the results obtained here.

Let us consider a system of N interacting relativistic particles. Let m_i ($i = 1, 2, \ldots, N$) be the mass of particle i, c is the speed of light and λ_i a Poincaré invariant parameter labeling the events along the world line $z_i^\mu(\lambda_i)$ of particle i in Minkowski spacetime. We denote $\dot{z}_i^\mu = \frac{dz_i^\mu}{d\lambda_i}$.

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The metric tensor: $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. We can write the Poincaré invariants [3,4]

$$\zeta_i = \dot{z}_i^2, \tag{1}$$

$$\xi_{ij} = (\dot{z}_i \dot{z}_j), \tag{2}$$

$$\gamma_{ij} = \big(\dot{z}_i (z_j - z_i)\big), \tag{3}$$

$$\rho_{ij} = (z_i - z_j)^2. \tag{4}$$

Let us consider the action

$$S = \sum_i m_i c \int d\lambda_i\, \zeta_i + \sum_i \sum_{j \neq i} \frac{g_i g_j}{c} \iint d\lambda_i\, d\lambda_j\, F(\xi_{ij}, \gamma_{ij}, \gamma_{ji}, \zeta_i, \zeta_j) \delta(\rho_{ij}). \tag{5}$$

The Dirac delta function in (5) accounts for the interactions propagating at the speed of light forward and backward in time. Without loss of generality, we assume the function F to be symmetric:

$$F(\xi_{ij}, \gamma_{ij}, \gamma_{ji}, \zeta_i, \zeta_j) = F(\xi_{ij}, \gamma_{ji}, \gamma_{ij}, \zeta_j, \zeta_i). \tag{6}$$

The Minkowski equations of motion for N interacting relativistic particles can be derived from the action (5) using the variational principle. We find[1]

$$m_i \ddot{z}_i^{\mu} = K_i^{\mu}, \tag{7}$$

where

$$K_i^{\mu} = \frac{g_i}{c^2} \sum_{j \neq i} g_j \int d\lambda_j\, \delta(\rho_{ij}) \big(A_{ij}^{\mu} + B_{ij}^{\mu\nu} \ddot{z}_{i\nu} + C_{ij}^{\mu\nu} \ddot{z}_{j\nu}\big), \tag{8}$$

$$A_{ij}^{\mu} = \frac{\partial F}{\partial z_{i\mu}} - \frac{\partial^2 F}{\partial z_i^{\eta} \partial \dot{z}_{i\mu}} \dot{z}_i^{\eta} + \frac{\zeta_j}{\gamma_{ji}^2} \left(\big(z_i^{\mu} - z_j^{\mu}\big) F + \gamma_{ij} \frac{\partial F}{\partial \dot{z}_{i\mu}} \right) + \frac{1}{\gamma_{ji}} \left(-\dot{z}_j^{\mu} F + \big(z_i^{\mu} - z_j^{\mu}\big) \frac{\partial F}{\partial z_j^{\eta}} \dot{z}_j^{\eta} + \xi_{ij} \frac{\partial F}{\partial \dot{z}_{i\mu}} + \gamma_{ij} \frac{\partial^2 F}{\partial z_j^{\eta} \partial \dot{z}_{i\mu}} \dot{z}_j^{\eta} \right), \tag{9}$$

$$B_{ij}^{\mu\nu} = -\frac{\partial^2 F}{\partial \dot{z}_{i\mu} \partial \dot{z}_{i\nu}}, \tag{10}$$

$$C_{ij}^{\mu\nu} = \frac{(z_i^{\mu} - z_j^{\mu})}{\gamma_{ji}} \left(\frac{\partial F}{\partial \dot{z}_{j\nu}} - \frac{(z_i^{\nu} - z_j^{\nu})}{\gamma_{ji}} F \right) + \frac{\gamma_{ij}}{\gamma_{ji}} \left(\frac{\partial^2 F}{\partial \dot{z}_{i\mu} \partial \dot{z}_{j\nu}} - \frac{(z_i^{\nu} - z_j^{\nu})}{\gamma_{ji}} \frac{\partial F}{\partial \dot{z}_{i\mu}} \right). \tag{11}$$

In order to identify λ_i with $s_i = c\tau_i$, where τ_i is the particle's proper time in flat spacetime, one needs to impose the well-known conditions:

$$(K_i \dot{z}_i) = 0. \tag{12}$$

The conditions (12) guarantee that, for all solutions of the equations of motion, ζ_i $(i = 1, 2, \ldots, N)$ are constants (which by simple scaling can be made equal to 1):

$$d\lambda_i^2 = \eta_{\mu\nu}\, dz_i^{\mu}\, dz_i^{\nu}. \tag{13}$$

Taking into account Eqs. (8) and (9)–(11) we can see that Eqs. (12) lead to the following conditions on F:

$$\gamma_{ji} \dot{z}_{i\eta} \frac{\partial \tilde{F}}{\partial z_{i\eta}} - \gamma_{ij} \dot{z}_{j\eta} \frac{\partial \tilde{F}}{\partial z_{j\eta}} - \left(\xi_{ij} + \zeta_j \frac{\gamma_{ij}}{\gamma_{ji}} \right) \tilde{F} = 0, \tag{14}$$

$$\frac{\partial \tilde{F}}{\partial \dot{z}_{i\nu}} = 0, \tag{15}$$

$$\big(z_i^{\nu} - z_j^{\nu}\big) \tilde{F} - \gamma_{ji} \frac{\partial \tilde{F}}{\partial \dot{z}_{j\nu}} = 0, \tag{16}$$

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[1] Equations (7)–(11) are derived from (5) by varying $z_{i\mu}$ in the action and integrating by parts, taking into account that

$$\frac{d}{d\lambda_i} \big(\delta(\rho_{ij})\big) = \frac{\frac{d\rho_{ij}}{d\lambda_i}}{\frac{d\rho_{ij}}{d\lambda_j}} \frac{d}{d\lambda_j} \big(\delta(\rho_{ij})\big) = \frac{\gamma_{ij}}{\gamma_{ji}} \frac{d}{d\lambda_j} \big(\delta(\rho_{ij})\big).$$