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Generalized tri-bimaximal neutrino mixing and its sensitivity to radiative corrections

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Abstract

We argue that the tri-bimaximal neutrino mixing pattern V_0 or its generalized form V_0', which includes two arbitrary Majorana phases of CP violation, may result from an underlying flavor symmetry at a superhigh energy scale close to the seesaw scale ($\sim 10^{14}$ GeV). Taking the working assumption that three neutrino masses are nearly degenerate, we calculate radiative corrections to V_0 and V_0' in their evolution down to the electroweak scale ($\sim 10^2$ GeV). Three mixing angles of V_0 or V_0' are essentially stable against radiative corrections in the standard model (SM). In the minimal supersymmetric standard model (MSSM), however, V_0 is in general disfavored and V_0' can be compatible with current neutrino oscillation data if its two Majorana phases α_1 and α_2 are properly fine-tuned. We also find that it is possible to radiatively generate the CP-violating phase δ from α_1 and α_2, and δ may keep on staying at its quasi-fixed point in either the SM or the MSSM.

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1. Current solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have provided us with very convincing evidence for the existence of neutrino oscillations, a quantum phenomenon which can naturally occur if neutrinos are massive and lepton flavors are mixed. The property of lepton flavor mixing can be described by a 3×3 unitary matrix V. A parametrization of V, advocated by the Particle Data Group [5], reads as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 23$ and 13). The phase parameters α_1 and α_2 are usually referred as to the Majorana CP-violating phases, because they have nothing to do with CP or T violation in the neutrino–neutrino and antineutrino–antineutrino oscillations. A global analysis of the present experimental data yields [6] $30° \leqslant \theta_{12} \leqslant 38°$, $36° \leqslant \theta_{23} \leqslant 54°$ and $0° \leqslant \theta_{13} \leqslant 10°$ as well as $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.2\text{–}8.9) \times 10^{-5}$ eV^2 and $\Delta m^2_{32} \equiv m^2_3 - m^2_2 = \pm(1.7\text{–}3.3) \times 10^{-3}$ eV^2 at the 99% confidence level. In contrast, three phases of V are entirely unrestricted. A variety of new neutrino experiments are underway, not only to detect the smallest flavor mixing angle θ_{13} and the phase parameter δ, but also to constrain the Majorana phases α_1 and α_2.

To interpret the observed neutrino mass spectrum and the observed bilarge neutrino mixing pattern, many theoretical and phenomenological models have been proposed and discussed [7]. A category of models or ansätze have attracted some particular

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attention, because they can give rise to the so-called tri-bimaximal neutrino mixing pattern [8]:

$$V_0 = \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix}. \tag{2}$$

Comparing between Eqs. (1) and (2), one may immediately observe that V_0 has $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ and $\alpha_1 = \alpha_2 = 0^\circ$. The phase parameter δ is not well defined in V_0, as a consequence of $\theta_{13} = 0^\circ$. The results $\sin^2 2\theta_{12} = 8/9$ and $\sin^2 2\theta_{23} = 1$ are in good agreement with current data on solar and atmospheric neutrino oscillations. It is straightforward to generalize V_0 in order to include two arbitrary Majorana phases,

$$V_0' = \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Although V_0 and V_0' have the same impact on neutrino oscillations, their consequences on the neutrinoless double-beta decay are certainly different. In this sense, we refer to V_0' as the *generalized* tri-bimaximal neutrino mixing pattern.

Such a special neutrino mixing pattern is in general expected to result from an underlying flavor symmetry (e.g., the discrete non-Abelian symmetry A_4 [9–12]) and its spontaneous or explicit breaking. The latter is always necessary, because a flavor symmetry itself cannot reproduce the observed lepton mass spectra and predict the realistic lepton mixing pattern simultaneously [13]. Specific and compelling constructions of this kind of flavor symmetry breaking are a real challenge and have been lacking, although some attempts have been made in the literature [7]. The energy scale, at which a proper flavor symmetry can be realized, may be considerably higher than the electroweak scale ($\Lambda_{\rm EW} \sim 10^2$ GeV). This new physics (NP) scale $\Lambda_{\rm NP}$ has actually been identified with other known scales in some model-building works [7], including the grand-unification-theory scale ($\Lambda_{\rm GUT} \sim 10^{16}$ GeV) or the seesaw scale ($\Lambda_{\rm SS} \sim 10^{14}$ GeV). In this case, radiative corrections to the relevant model parameters between $\Lambda_{\rm EW}$ and $\Lambda_{\rm NP}$ must be taken into account [14].

One may then ask whether the generalized tri-bimaximal neutrino mixing pattern is stable or not against radiative corrections, if it is derived from an underlying (broken) flavor symmetry within an unspecified mechanism at $\Lambda_{\rm NP}$ ($\gg \Lambda_{\rm EW}$). The main purpose of this Letter is just to answer this question by considering both the standard model (SM) and its minimal supersymmetric extension (MSSM) below $\Lambda_{\rm NP}$. The only effective dimension-5 operator of light Majorana neutrinos reads as

$$\mathcal{L}_\nu = \frac{1}{2}\kappa_{ij}(H \cdot L_i)(H \cdot L_j) + \text{h.c.}, \tag{4}$$

where H denotes the SM Higgs (or the MSSM Higgs with the appropriate hypercharge), L_i (for $i = 1, 2, 3$) stand for the leptonic $SU(2)_{\rm L}$ doublets, and κ is a symmetric neutrino coupling matrix. After spontaneous gauge symmetry breaking at $\Lambda_{\rm EW}$, we arrive at the effective neutrino mass matrix $M_\nu = v^2\kappa$ (SM) or $M_\nu = v^2\kappa \sin^2\beta$ (MSSM), where $v \approx 174$ GeV and $\tan\beta$ is the ratio of the vacuum expectation values of two Higgs fields in the MSSM. Between $\Lambda_{\rm EW}$ and $\Lambda_{\rm NP}$, the most important radiative correction to κ is proportional to $\ln(\Lambda_{\rm NP}/\Lambda_{\rm EW})$ and can be evaluated by using the one-loop renormalization group equations (RGEs) [14]. It is then possible to calculate the RGE effects on the lepton flavor mixing parameters analytically and numerically.

In the working assumption that three neutrino masses are nearly degenerate, we are going to calculate radiative corrections to V_0 and V_0'. We show that both V_0 and V_0' are stable against radiative corrections in the SM, but only V_0' with the proper fine-tuning of $(\alpha_2 - \alpha_1)$ is allowed in the MSSM. In addition, the CP-violating parameter δ can be radiatively generated from α_1 and α_2. A peculiar feature of δ is that it may keep on staying at its quasi-fixed point in both the SM and the MSSM.

2. Taking account of the seesaw mechanism [15] as a natural idea to understand the origin of neutrino masses and lepton flavor mixing, we assume the new physics (i.e., new flavor symmetry) scale $\Lambda_{\rm NP}$ is close to the seesaw scale $\Lambda_{\rm SS} \sim 10^{14}$ GeV. Below $\Lambda_{\rm NP}$, the effective neutrino coupling matrix κ obeys the one-loop RGE [16][2]:

$$16\pi^2 \frac{{\rm d}\kappa}{{\rm d}t} = \alpha\kappa + C\left[\left(Y_l Y_l^\dagger\right)\kappa + \kappa\left(Y_l Y_l^\dagger\right)^T\right], \tag{5}$$

in which $t \equiv \ln(\mu/\Lambda_{\rm NP})$ with μ being an arbitrary renormalization scale below $\Lambda_{\rm NP}$ but above $\Lambda_{\rm EW}$. In the SM, $C = -1.5$ and $\alpha \approx -3g_2^2 + 6y_t^2 + \lambda$; and in the MSSM, $C = 1$ and $\alpha \approx -1.2g_1^2 - 6g_2^2 + 6y_t^2$, where g_1 and g_2 denote the gauge couplings, y_t stands for the top-quark Yukawa coupling, and λ represents the Higgs self-coupling in the SM [16]. In the flavor basis where the charged-lepton Yukawa coupling matrix is diagonal and real (positive), we have $\kappa = V\bar{\kappa}V^T$ with $\bar{\kappa} = {\rm Diag}\{\kappa_1, \kappa_2, \kappa_3\}$. The

[2] Note that $\Lambda_{\rm NP} \sim \Lambda_{\rm SS}$ is an effective working assumption, in which the possible mass hierarchy of three heavy right-handed neutrinos N_i (for $i = 1, 2, 3$) is omitted. If $\Lambda_{\rm NP} \sim \Lambda_{\rm GUT}$ ($> \Lambda_{\rm SS}$) is assumed and the mass hierarchy of N_i is considered, then very strong seesaw threshold effects may appear in the RGE evolution of relevant model parameters (see Ref. [17] for detailed discussions).