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Radiative corrections in a vector-tensor model

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Abstract

In a recently proposed model in which a vector non-Abelian gauge field interacts with an antisymmetric tensor field, it has been shown that the tensor field possesses no physical degrees of freedom. This formal demonstration is tested by computing the one-loop contributions of the tensor field to the self-energy of the vector field. It is shown that despite the large number of Feynman diagrams in which the tensor field contributes, the sum of these diagrams vanishes, confirming that it is not physical. Furthermore, if the tensor field were to couple with a spinor field, it is shown at one-loop order that the spinor self-energy is not renormalizable, and hence this coupling must be excluded. In principle though, this tensor field does couple to the gravitational field.

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The antisymmetric tensor field $\phi_{\mu\nu}$ has been examined for some time [1–5]. Generally, it is considered to have a local Abelian gauge invariance $\phi_{\mu\nu} \to \phi_{\mu\nu} + \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu$. This is consistent with coupling to the field strength $F^{\mu\nu}$ associated with a vector gauge field A^μ as $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\phi^{\mu\nu}F^{\lambda\sigma}$ is invariant under gauge transformations of both $\phi_{\mu\nu}$ and A^μ. This coupling can be used to provide a mass to the Abelian vector field A^μ without destroying gauge invariance or introducing a Higgs scalar. However, a non-Abelian generalization of this mechanism of mass generation that is consistant with renormalizability has proved to be elusive [6].

Recently, a model has been considered in which a non-Abelian gauge field W^a_μ interacts with an antisymmetric tensor field $\phi^a_{\mu\nu}$ with the Lagrange density [7]

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{12}G^a_{\mu\nu\lambda}G^a_{\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\phi^a_{\mu\nu}F^a_{\lambda\sigma} + \frac{\mu^2}{8}\epsilon_{\mu\nu\lambda\sigma}\phi^a_{\mu\nu}\phi^a_{\lambda\sigma}. \tag{1}$$

In Eq. (1), m and μ are mass parameters,

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc}W^b_\mu W^c_\nu, \tag{2}$$

$$G^a_{\mu\nu\lambda} = D^{ab}_\mu \phi^b_{\nu\lambda} + D^{ab}_\nu \phi^b_{\lambda\mu} + D^{ab}_\lambda \phi^b_{\mu\nu} \tag{3}$$

and

$$D^{ab}_\mu = \partial_\mu \delta^{ab} + gf^{apb}W^p_\mu. \tag{4}$$

Even in the Abelian limit, the mass term in Eq. (1) that is bilinear in $\phi_{\mu\nu}$ is not gauge invariant, and so it was originally hoped that inclusion of this term would provide a mass to the vector field W_μ that would have a non-Abelian generalization.

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This Lagrange density is invariant under the infinitesimal gauge transformation

$$\delta W^a_\mu = D^{ab}_\mu \Omega^b, \qquad \delta\phi^a_{\mu\nu} = g f^{abc}\phi^b_{\mu\nu}\Omega^c. \tag{5}$$

Both by a canonical analysis using the Dirac constraint formalism [8] and by explicit eliminations of non-physical degrees of freedom, it has been shown that surprisingly in the Abelian limit, the tensor field in Eq. (1) does not possess any physical degrees of freedom.

It was surmised that the full non-Abelian model of Eq. (1) also does not contain any physical degrees of freedom associated with the tensor field; this conjecture is what we can test by an explicit calculation. Evaluation of the one-loop contributions to the vector self-energy $\langle W^a_\mu W^b_\nu\rangle$ involves Feynman diagrams with both vertices and propagators associated with the tensor field, and if the tensor field is indeed non-physical, its contributions to this Green's function should all cancel. Below we show that this is in fact what happens.

Working in Euclidean space, the contribution to L in Eq. (1) that is bilinear in the fields is

$$L^{(2)} = \frac{1}{2}\left(W^a_\lambda, \phi^a_{\alpha\beta}\right)\begin{pmatrix} \partial^2 I_{\lambda\sigma} & \frac{m}{2}B_{\lambda,\gamma\delta} \\ \frac{m}{2}A_{\alpha\beta,\sigma} & -\frac{1}{2}I_{\alpha\beta,\gamma\delta}\partial^2 + Q_{\alpha\beta,\gamma\delta} + \frac{\mu^2}{4}\epsilon_{\alpha\beta\gamma\delta}\end{pmatrix}\begin{pmatrix} W^a_\sigma \\ \phi^a_{\gamma\delta}\end{pmatrix}, \tag{6}$$

where

$$\begin{aligned}
&I_{\alpha\beta} = \delta_{\alpha\beta}, \qquad L_{\alpha\beta} = \partial_\alpha\partial_\beta, \qquad I_{\alpha\beta,\gamma\delta} = \frac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}),\\
&A_{\mu\nu,\lambda} = \epsilon_{\mu\nu\kappa\lambda}\partial_\kappa = -B_{\lambda,\mu\nu}, \qquad C_{\mu\nu,\lambda} = \delta_{\mu\lambda}\partial_\nu - \delta_{\nu\lambda}\partial_\mu = D_{\lambda,\mu\nu},\\
&Q_{\mu\nu,\lambda\sigma} = \frac{1}{4}\left(\delta_{\mu\lambda}\partial^2_{\nu\sigma} + \delta_{\nu\sigma}\partial^2_{\mu\lambda} - \delta_{\mu\sigma}\partial^2_{\nu\lambda} - \delta_{\nu\lambda}\partial^2_{\mu\sigma}\right) \quad \left(\partial^2_{\alpha\beta} \equiv \frac{\partial^2}{\partial x^\alpha\,\partial x^\beta}\right),\\
&L_{\mu\nu,\lambda\sigma} = \epsilon_{\mu\nu\kappa\lambda}\partial^2_{\kappa\sigma} - \epsilon_{\mu\nu\kappa\sigma}\partial^2_{\kappa\lambda} = -R_{\lambda\sigma,\mu\nu}.
\end{aligned} \tag{7}$$

We have used a gauge fixing Lagrangian

$$L_{gf} = -\frac{1}{2}\left(\partial\cdot W^a\right)^2. \tag{8}$$

The inverse of the matrix M appearing in Eq. (6) is

$$M^{-1} = \begin{pmatrix} \frac{I_{\sigma\kappa}}{\partial^2} & -\left(\frac{m}{\mu^2\partial^2}\right)D_{\sigma,\pi\tau} \\ \left(\frac{m}{\mu^2\partial^2}\right)C_{\gamma\delta,\kappa} & \left(\frac{4}{\mu^4}\right)\left(1-\frac{m^2}{\partial^2}\right)Q_{\gamma\delta,\pi\tau} + \left(\frac{1}{\mu^2\partial^2}\right)\left(-L_{\gamma\delta,\pi\tau} + R_{\gamma\delta,\pi\tau}\right)\end{pmatrix}. \tag{9}$$

(This corrects a minor mistake in Ref. [7].)

Using Eq. (9), the free field propagators can be determined. The Feynman rules needed to determine the contribution of the tensor field to $\langle W^a_\mu W^b_\nu\rangle$ are in Fig. 1.

In computing the one-loop corrections to $\langle W^a_\mu W^b_\nu\rangle$, it is necessary to include diagrams in which the external leg involves the mixed propagator $\langle W^a_\mu \phi^b_{\lambda\sigma}\rangle$.

The presence of the tensor $\epsilon_{\mu\nu\lambda\sigma}$ in the Lagrange density of Eq. (1) makes straightforward application of dimensional regularization difficult. The aspects of dimensional regularization needed are that shifts of variables of integration in Feynman integrals do not generate surface terms, that massless tadpole integrals of the form $\int \frac{d^nk}{(2\pi)^n}\frac{1}{(k^2)^a}$ vanish, and that

$$\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\lambda\sigma} = (\delta_{\alpha\mu}\delta_{\beta\nu}\delta_{\gamma\lambda}\delta_{\delta\sigma} + \cdots), \tag{10}$$

where in Eq. (10) all 24 terms formed by permuting indices (taking into account the antisymmetry of $\epsilon_{\alpha\beta\gamma\delta}$) are taken into account. If there were to be modifications to Eq. (10) by virtue of working in $(n-4)$ dimensions, then these corrections would be of order $(n-4)$. This would result in finite contributions to the two point function if there were any divergent integrals, but as the Feynman integrals all cancel, any corrections to Eq. (10) can be ignored. We then use the n-dimensional relations $\delta_{\mu\mu} = n$ and

$$\int \frac{d^nk}{(2\pi)^n}k_\mu k_\nu f\left(k^2\right) = \frac{1}{n}\delta_{\mu\nu}\int \frac{d^nk}{(2\pi)^n}k^2 f\left(k^2\right). \tag{11}$$

It turns out though that no integral over loop momentum has to be performed.

The Feynman diagrams associated with the one-loop corrections to $\langle W^a_\mu W^b_\nu\rangle$ involving the tensor field all vanish individually except for the ones of Fig. 2. These are individually non-zero, but their sum reduces to an integral of the form

$$\Pi_{\mu\nu}(p) = \int \frac{d^nk}{(2\pi)^n}\int_0^1 dx\,(1-2x)\,f\left(x(1-x), k^2, p^2\right)\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right). \tag{12}$$