Composites Science and Technology 113 (2015) 19–30

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

Novel composites with asymmetrical elastic wave properties

Lang-Quan Shui *, Zhu-Feng Yue, Yong-Shou Liu, Qing-Chang Liu, Jiao-Jiao Guo, Xin-Dang He

Department of Engineering Mechanics of Northwestern Polytechnical University, Xi'an 710129, China

ARTICLE INFO

Article history:
Received 26 June 2014
Received in revised form 7 February 2015
Accepted 10 March 2015
Available online 19 March 2015

Keywords:
A. Structural composites
A. Smart materials
B. Mechanical properties
Asymmetrical elastic wave

ABSTRACT

The traditional structural composites are known as materials with a spatial order. The temporal order is introduced into structural composites in this paper. We theoretically describe and numerically demonstrate the architecture for a structural composite which exhibits tunable and nearly "full-banded" asymmetrical elastic wave property. Given its spatiotemporal order and unconventional wave properties, such composite can be named as spatiotemporal metamaterial (STMM). Firstly, the effective material parameters, which include some new material parameters, are obtained based on improved multi-scale homogenization techniques. Secondly, the unconventional wave properties are theoretically analyzed. It is found that the elastic wave group velocity surfaces do not satisfy the centrosymmetry in STMM, i.e. the time-reversal symmetry of linear elastic wave is violated. In certain instances, the waves even propagate unidirectionally. Finally, we present an example of the STMMs, which includes homogenized calculations and numerical simulations where the shear waves are forbidden in some directions. The unconventional wave properties of the STMMs proposed in this paper own attractive potential applications, such as acoustic absorbing, acoustic information processing, energy-saving, and energy-harvesting.

1. Introduction

Metamaterials are artificially engineered composite structures that exhibit unconventional wave properties. Originating from the microwave technology [1], unprecedented applications of metamaterials have been expanded from optics to mechanics, acoustics, thermodynamics and other fields [2], which makes this research topic become one of the fastest growing subdisciplines of material science [3].

While optics and acoustic cloaking are attractive applications of metamaterials, the study on metamaterials bears far more significance. Leonhardt [4] claimed that perfect geometrical optical invisibility could be possible when using modern metamaterials, and also pointed out that the corresponding method could be applied to other electromagnetic waves or sound. Alù and Engheta [5] discussed the possibility of using plasmonic and metamaterial covers to make spherical and cylindrical objects nearly "invisible". Zhang et al. [6] presented the first practical realization of a metamaterial cloak for underwater ultrasound. Popa et al. [7] proposed a class of two-dimensional acoustic cloakings, which is easily achievable in practice by using metamaterials. Broadly speaking, the structure with asymmetrical wave properties belongs to metamaterials [8]. Achieving asymmetrical wave properties has become one of the hot topics in the metamaterial research. The work of Terraneo et al. [9] initiated the possibility to propose a thermal rectifier. Wang and Li [10] established thermal logic gates and discussed the possibility of nanoscale experiments. Fan et al. [11] demonstrated a passive optical diode with a high transmission ratio based on optical nonlinearity, which might lead to a revolution in modern information processing. Liang et al. [12,13] demonstrated a one-dimensional acoustic diode and achieved acoustic rectification, in which it is emphasized that such a device could be applied in biomedicine. Nesterenko et al. [14] and Liang indicated the applications of acoustic diodes in the areas of energy-saving and energy-harvesting. Boechler et al. [15] demonstrated a new mechanism for tunable rectification, which uses bifurcations and chaos, in which the mechanism was claimed to be applicable to design advanced photonic, thermal and acoustic materials and devices. Lepri and Casati [16] designed a diode chain in which the waves propagate with the same amplitude but different frequencies in opposite directions. In addition, Silva et al. [17] introduced the concept of metamaterial analog computing and pointed out that properly designed metamaterials can perform mathematical operations. Mei et al. [18] presented a thin-film acoustic metamaterial which can completely absorb the selected low-frequency airborne sound. Popa et al. [19] demonstrated an acoustic metamaterial with effective tunable material parameters including negative material

* Corresponding author. Tel.: +86 15902902159.
E-mail address: shuilq061341@gmail.com (L.-Q. Shui).

http://dx.doi.org/10.1016/j.compscitech.2015.03.007
0266-3538/

parameters. In general, numerous studies about metamaterials have been published as far.

Despite significant progress, metamaterials still face challenges, such as passivity, narrowband and high loss [19,20]. The desired metamaterials should be fully dynamic, broad bandwidth, loss-free and truly 3D [3,21,22]. Inspired by the work of Li et al. [23], Wilczek [24], Shapere and Wilczek [25] and Bergamini et al. [26] (see Section 2), a temporal order is introduced into metamaterials, which shows surprisingly promising outcome.

Specifically, an architecture is proposed for a structural composite with time-varying unit cells. With some new material parameters describing the mechanical properties, the effective material parameters are obtained based on improved multi-scale homogenization techniques. The result reveals that the coupling of the spatial and temporal order leads to asymmetric wave phenomenon. In such material, the elastic waves propagate with different group velocities in opposite directions and even propagate unidirectionally, i.e. the time-reversal symmetry is violated. Obviously, its group velocity surfaces do not satisfy centrosymmetry. With the time-reversal asymmetry, such a wave property could be named as spatiotemporal anisotropy (STA). With these unconventional wave properties, the material might be proposed for applications in acoustic absorbing, acoustic information processing, energy-saving, energy-harvesting and so on [14].

Considering the spatiotemporally ordered mesoscopic structure, and unconventional wave properties in such a material, we name it STMM. The temporal variability of the unit cells can be achieved by using the materials with controllable properties (e.g. smart materials. See Section 2), which fundamentally enable the tunability of metamaterials. Thus, the wave properties can be modified artificially. In addition, the homogenization techniques require much longer wavelength with respect to the scale of the unit cell, indicating that the wave frequency would fall into any desirable range if the unit cells are properly small. Overall, the STMM would be a tunable and nearly "full-banded" metamaterial.

2. The proposal of STMMs

First, we shall mention the work that inspired us for this proposal. Li et al. [23], Wilczek [24], Shapere and Wilczek [25] proposed and explored the concept of crystals with spatial and temporal order in 2012, respectively. Coleman [27] highly praised their work and pointed out that their work might initiate a brand new field of research. Bergamini et al. [26] proposed a phononic crystal with adaptive connectivity in 2013. This phononic crystal might involve an embryonic idea of STMM. In this phononic crystal, the unit cell has locally controlled connectivity. Bergamini claimed that a fairly small amount of adaptive material can produce remarkable effects due to the exploitation of system periodicity.

In parallel, we propose an architecture with time-varying unit cells by introducing the temporal order into metamaterials. The temporal variability of the unit cells can be achieved by controllable properties of the materials. Such materials can be the piezoelectric material and electro-rheological (ER) materials experiencing time-varying electric field [28,29], the magnetostrictive material and magneto-rheological fluid experiencing time-varying magnetic field [30,31], and the shape-memory material experiencing time-varying temperature field [32]. Popa et al. [19] pointed out that the active response of the controllable properties can be instantaneously achieved because of low group velocities of the mechanical waves. The so-called "time-varying unit cells" may be abstract and contrary to the conventional cognition. For a better understanding of the architecture, the implementation of a STMM in Section 5 and some general analyses of time-varying property in Appendix A can be referred to. Finally, we want to point out that the controllability of the unit cells fundamentally enables the tunability of the STMM.

3. The dynamic multi-scale homogenization in STMMs

In the exploration of the material property of structural composites, the elastic wave whose wavelength is much longer than the unit cell scale is considered. To obtain the macroscopic constitutive relations of an architecture, the effective material parameters should be calculated. It is well known that computational homogenization is a particularly suitable technique to calculate the effective material parameters of structural composites.

Computational homogenization, as a powerful technique to solve partial differential equations with rapidly oscillating coefficients, has been widely applied in micromechanics [33–36]. In STMMs, the meso-structure exhibits a spatial and temporal order. Consequently, the corresponding elastic wave equations have rapidly oscillating coefficients with spatial and temporal periodicity. Casado-Diaz et al. [37] have proved the feasibility to apply the multi-scale homogenization technique to such wave equations. Dong and Cao [38] have also applied the technique to similar equations.

A linearized Hookean body is considered in this paper. Let $u_i(x_1, x_2, x_3, t)$, $i = 1, 2, 3$, describe the displacement of a particle located at (x_1, x_2, x_3) at time t from its position in the natural state. The infinitesimal strain tensor is expressed as

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1}$$

In a small deformation framework, the particle velocity can be expressed as

$$v_i = \frac{\partial u_i}{\partial t} \tag{2}$$

The corresponding stress and momentum are expressed as

$$\sigma_{ij} = \lambda_{ijmn} e_{mn} \tag{3}$$

$$p_i = \rho v_i \tag{4}$$

where λ_{ijkl} is the adiabatic elastic tensor which satisfies Voigt symmetry, and ρ is the density. The elastic tensor and density are periodic functions of space and time. For isotropic materials, the elastic tensor $\lambda_{ijmn} = \lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})$, where λ and μ are Lame constants, and δ_{ij} is the Kronecker symbol.

The wave equation of the linearized Hookean body is written as

$$\frac{\partial p_i}{\partial t} + f_i = \frac{\partial \sigma_{ij}}{\partial x_j} \tag{5}$$

where f_i is the volume force. Based on Eqs. (1)–(5), the wave equation can be expressed as

$$\frac{\partial}{\partial t}\left(\rho \frac{\partial u_i}{\partial t}\right) + f_i = \frac{\partial}{\partial x_j}\left(\lambda_{ijmn}\frac{\partial u_m}{\partial x_n}\right) \tag{6}$$

The homogenization of Eq. (6) usually begins with separation of variables [25,39]. The frequency invariance makes it possible to separate the temporal variable and spatial variables. However, for the material we proposed, the time-varying property breaks the frequency invariance (Appendix A). Thus we adopt new ideas to homogenize Eq. (6), where the wave equation can be rewritten as