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Theory of the Lamb Shift and fine structure in muonic ^{4}He ions and the muonic ^{3}He–^{4}He Isotope Shift

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ABSTRACT

We provide an up to date summary of the theory contributions to the 2S → 2P Lamb shift and the fine structure of the 2P state in the muonic helium ion $(\mu^4\mathrm{He})^+$. This summary serves as the basis for the extraction of the alpha particle charge radius from the muonic helium Lamb shift measurements at the Paul Scherrer Institute, Switzerland. Individual theory contributions needed for a charge radius extraction are compared and compiled into a consistent summary. The influence of the alpha particle charge distribution on the elastic two-photon exchange is studied to take into account possible model-dependencies of the energy levels on the electric form factor of the nucleus. We also discuss the theory uncertainty which enters the extraction of the ^{3}He–^{4}He isotope shift from the muonic measurements. The theory uncertainty of the extraction is much smaller than a present discrepancy between previous isotope shift measurements. This work completes our series of $n = 2$ theory compilations in light muonic atoms which we have performed already for muonic hydrogen, deuterium, and helium-3 ions.

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Contents

1. Introduction

The CREMA Collaboration has measured both 2S → 2P Lamb shift transitions in the muonic helium ion $(\mu^4\text{He})^+$ [1,2]. A scheme of these energy levels in muonic helium-4 ions is shown in Fig. 1. In preparation of the upcoming extraction of nuclear properties from these measurements, such as the nuclear root-mean-square (rms) charge radius r_α, we provide a careful study of the available calculations of the theory contributions to the involved energy levels, summarizing the results of several theory groups.

Both, the Lamb shift and the fine structure, have been analyzed recently [3–6] but significant differences between the authors made it necessary to review the individual theory contributions. The same was previously done for muonic hydrogen [7], muonic deuterium [8], and muonic helium-3 ions [9].

Recent measurements of the 2S → 2P Lamb shift (LS) in other muonic atoms have already provided the rms charge radii of the proton and the deuteron with unprecedented precision. Results from muonic hydrogen measurements provided a proton charge radius of

$$r_p^\mu = 0.84087(26)^{\text{exp}}(29)^{\text{theo}}\ \text{fm}\ [10, 11]. \tag{1}$$

This value is ten times more precise than the CODATA-2014 value of 0.8751(61) fm [10], however also 4%, or 6 σ, smaller. This discrepancy created the so-called "Proton-Radius-Puzzle" (PRP) [11–15].

A recent determination of the deuteron radius from muonic deuterium spectroscopy results in a value of

$$r_d^\mu = 2.12616(13)^{\text{exp}}(89)^{\text{theo}}\ \text{fm}\ [18–20], \tag{2}$$

that is also smaller than the CODATA value and hints towards a change in the Rydberg constant [16,17], but note the recent result [18]. The value in Eq. (2) differs slightly from our published value of $r_d^\mu = 2.12562(13)^{\text{exp}}(77)^{\text{theo}}$ fm [19] due to updated nuclear theory of the two-photon contributions by Hernandez et al. [20] and the unexpectedly large three-photon contribution recently calculated for the first time by Pachucki et al. [21].

The determination of the alpha particle charge radius from muonic helium-4 ions, when compared to the radius determinations from electron scattering experiments [22,23] will provide new input on the existing discrepancies. The improved value of r_α will be used in the near future for tests of fundamental bound state quantum electrodynamics (QED) by measurements of the 1S → 2S transition in electronic He^+ ions [24,25]. Furthermore, the combination of the precise charge radii from muonic helium-3 and helium-4 ions will contribute to solving a discrepancy between several isotope