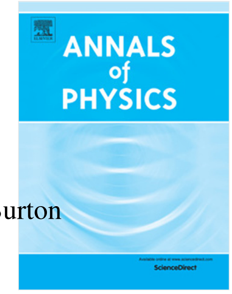

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PII: S0003-4916(18)30174-X
DOI: https://doi.org/10.1016/j.aop.2018.07.003
Reference: YAPHY 67698

To appear in: *Annals of Physics*

Received date : 17 June 2018
Accepted date : 2 July 2018

Coupled complex Ginzburg-Landau systems with saturable nonlinearity and asymmetric cross-phase modulation

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Abstract

We formulate and study dynamics from a complex Ginzburg-Landau system with saturable nonlinearity, including asymmetric cross-phase modulation (XPM) parameters. Such equations can model phenomena described by complex Ginzburg-Landau systems under the added assumption of saturable media. When the saturation parameter is set to zero, we recover a general complex cubic Ginzburg-Landau system with XPM. We first derive conditions for the existence of bounded dynamics, approximating the absorbing set for solutions. We use this to then determine conditions for amplitude death of a single wavefunction. We also construct exact plane wave solutions, and determine conditions for their modulational instability. In a degenerate limit where dispersion and nonlinearity balance, we reduce our system to a saturable nonlinear Schrödinger system with XPM parameters, and we demonstrate the existence and behavior of spatially heterogeneous stationary solutions in this limit. Using numerical simulations we verify the aforementioned analytical results, while also demonstrating other interesting emergent features of the dynamics, such as spatiotemporal chaos in the presence of modulational instability. In other regimes, coherent patterns including uniform states or banded structures arise, corresponding to certain stable stationary states. For sufficiently large yet equal XPM parameters, we observe a segregation of wavefunctions into different regions of the spatial domain, while when XPM parameters are large and take different values, one wavefunction may decay to zero in finite time over the spatial domain (in agreement with the amplitude death predicted analytically). We also find a collection of transient features, including transient defects and what appear to be rogue waves, while in two spatial dimensions we observe highly localized pattern formation. While saturation will often regularize the dynamics, such transient dynamics can still be observed - and in some cases even prolonged - as the saturability of the media is increased, as the saturation may act to slow the timescale.

Keywords: complex Ginzburg-Landau system, saturable nonlinearity, cross-phase modulation, modulational instability, spatiotemporal dynamics

1. Introduction

The complex Ginzburg-Landau (GL) equation models a variety of phenomena, including nonlinear waves, second-order phase transitions, superconductivity, superfluidity, Bose-Einstein condensation, liquid crystals, and strings in field theory [1]. The most common example is the cubic GL equation, for which a variety of solutions have been found [2]. Spatiotemporal chaos has been observed in the complex cubic GL equation [3, 4]. Transition to spatiotemporal turbulence from traveling waves due to instabilities was discussed in [5]. Spatial patterning is also possible, with solutions such as spiral waves being found [6]. Bounds on the attractor for complex cubic GL equations have been considered in [7, 8]. Quintic or cubic-quintic GL equations have been studied [9], and have application to areas of nonlinear optics including lasers [10], optical waveguides equipped with a Bragg grating [11], quasi-CW Raman fiber lasers [12], and Bose-Einstein condensates [13, 14]. Solutions to non-autonomous forms of the complex GL equation also have been studied [15].

Coupled or vector complex cubic GL equations have also attracted interest. For example, the effect of slow real modes in reaction-diffusion systems close to a supercritical Hopf bifurcation was studied in [16] via a system of two coupled GL equations, where it was pointed out that a single GL equation cannot reproduce the dynamics from the

Preprint submitted to Annals of Physics *June 17, 2018*