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Renormalons in a general Quantum Field Theory

Alessio Maiezza [a], Juan Carlos Vasquez [b,*]

[a] Ruđer Bošković Institute, Bijenička cesta 54, 10000, Zagreb, Croatia
[b] Universidad Técnica Federico Santa María & CCTVAL, Valparaíso, Chile

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ABSTRACT

We generalize the concept of Borel resummability and renormalons to a quantum field theory with an arbitrary number of fields and couplings, starting from the known notion based on the running coupling constants. An approach to identify the renormalons is provided by exploiting an analytic solution of the generic one-loop renormalization group equations in multi-field theories. Methods to evaluate the regions in coupling space where the theory is resummable are described. The generalization is then illustrated in a toy model with two coupled scalar fields, representing the simplest extension of the one-field analysis presented in the seminal works of the subject. Furthermore, possible links to realistic theories are briefly discussed.

1. Introduction

It has been known since the original argument given by Dyson [1], that the all the power series used in Quantum Field Theory (QFT) diverge as $n!$, where n is the order in the perturbative expansion. However, the convergence of a divergent series can be improved through the Borel transform, followed by an analytic continuation and finally coming back to the original variable and a finite output through the Laplace transform. This procedure is called resummation and helps to make sense of the divergent series in QFT. Unfortunately, even the Borel series may diverge in some cases [2–5]. This divergence may be due to the instantons [5], but these may be treated consistently by semiclassical methods [2,6–8].

The real problem is due to another type of divergence of the Borel series, the so-called renormalons [2], whose physics is not well understood. As suggested by the name first introduced by 't Hooft, the renormalons arise from the procedure of renormalization and bring in ambiguities in

* Corresponding author.
E-mail addresses: amaiezza@irb.hr (A. Maiezza), juan.vasquezcar@usm.cl (J.C. Vasquez).

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the perturbative formulation of the theory when the coupling is large enough (for complete reviews see [9–12]). Since a rigorous proof of the existence of renormalons has not yet been found, there is a claim that they may not even exist at all [13]. Nevertheless, in Ref. [14] the existence of renormalons was established for the N component ϕ^4 scalar field theory. Moreover, in Ref. [15] compelling evidence of the existence of the factorial growth predicted by renormalon analysis has also been found in QCD, where the renormalons have been used to estimate the non-perturbative power corrections [16,17].

In this work, we propose the generalization of the original concept of renormalons to a multi-field and multi-coupling framework starting from the argument presented in [2] and connected with the renormalization group in [3,4], the latter offering a natural way to extend it through the notion of the running couplings. The enlargement to multi-field theories in non-trivial in several aspects. A relevant obstacle is to obtain the analytic solution of renormalization-group-equations [18] (RGEs). A coupled system of differential equations is in general not solvable analytically. We circumvent the problem by exploiting an iterative solution of the RGEs. This enables us to define a multi-variable Borel and Laplace transforms, to estimate the singularity of the former and to identify the renormalons related to the ambiguity of the latter. Particular attention is paid to scalar field theories. In order to illustrate the proposed method, we study a simple prototype model of two coupled scalar fields, the minimal generalization of the example in Ref. [2].

The article is organized as follows: in Section 2, for the reader's ease, we review the main features of the singularities of the Borel transform with a particular focus on renormalons. In Section 3 we show the recursive and analytical solution for the RGEs. Then in Section 4 we construct the general Borel transform and show how to estimate its singularities, within different approximations, analytically or numerically. In Section 5 and for the sake of illustration, we apply the method previously discussed for a scalar toy model. In Section 6, we comment on the contact with realistic models and conclusions are given in Section 7. Finally, the paper is equipped with several Appendices for further details.

2. Renormalons: main features

In this section we review the main features of the singularities of the Borel transform, known as instantons and renormalons, disentangling them and focusing on the latter. The aim is to define a starting point and a road-map for the generalization of renormalons in any QFT through the Borel resummability in multi-variables series.

2.1. Divergences of Borel series: instantons vs renormalons

The procedure of resummation improves the convergence of the perturbation series in QFT. This is achieved through the Borel transform of those series (see Ref. [19] for the mathematical theory of resurgent analysis). To be more explicit and to provide a set-up, it may be useful to start considering the over-simplistic case of a QFT in one space–time point [2], so that the functional integral reduces to

$$G(\lambda) = \int dx e^{-\frac{1}{2}x^2 - \frac{\lambda}{4!}x^4} . \tag{1}$$

This function can be re-written as a Laplace transform

$$G(\lambda) = \int_0^\infty dz F(z) e^{-z/\lambda} , \tag{2}$$

where $F(z)$ is the Borel transform. Thus the Laplace transform is well-defined if $F(z)$ has no poles on the real and positive axes, otherwise, the usual perturbative approach to the theory becomes ambiguous. The ambiguity is due to the two possible contour deformations that one may choose in order to avoid the pole in the positive real axes when applying the inverse Borel transform. An insight regarding the position of the pole in $F(z)$ can be obtained by writing it as

$$F(z) = \int dx \delta(z - \lambda S(x)) , \tag{3}$$