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Novel results for kinklike structures and their connections to quantum mechanics

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ABSTRACT

In this work we use the deformation procedure and explore the route to obtain distinct field theory models that present similar stability potentials. Starting from systems that interact polynomially or hyperbolically, we use a deformation function which allows to construct other theories having the feature of giving rise to the same stability potentials. Such deformation function leads to smooth potentials according to a specific choice of a single parameter. Among the results, one shows that for models with asymmetric topological sectors, the appearance of a new stability potential is also possible.

1. Introduction

Topological defects are localized structures that appear in a diversity of contexts [1–5] in high energy physics. Among the most popular, stand out the one-dimensional structures called kinks which may provide important information about the behavior of several physical systems; see, e.g., Refs. [1–12]. In the case of field theories, such structures appear generally as solutions of nonlinear equations of motion and present localized energy density and finite energy.

An interesting fact about kinks concerns their linear stability, whose analysis leads to a direct connection with supersymmetry and quantum mechanics [13,14]. This connection is an old issue and has been investigated in several works, in particular in Refs. [15–24]. Along the years, it has been shown that the subject engenders two interesting routes, one that goes from field theory to quantum mechanics, when one investigates stability of the kinklike structures in a field theory model, which

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leads us to a stability potential that simulates a quantum mechanical potential. The other route is referred to as the reconstruction procedure, in which a field theory model is constructed from specific features of a given quantum mechanical potential, in particular its zero or translational mode. In the works [20,21] the authors presented interesting investigations concerning the reconstruction of field theories from the spectrum of quantum mechanical potentials, and these studies have motivated the recent Refs. [22–24] to further explore the subject, bringing novel results. For instance, in Ref. [22] it was shown that the same scalar field theory may describe distinct quantum mechanical problems, and in Ref. [23] it was shown that the reconstruction of a field theory model is not unique, that is, we can reconstruct distinct field theories from the very same quantum mechanical potential. These results are of current interest and more recently yet, the investigation in [24] offered the possibility of constructing distinct field theory models with the very same stability potential, thus leading us to the same quantum mechanical potential.

The current work is aimed to enlarge and deepen the results obtained in Refs. [22–24]. In this sense, we use the deformation procedure [25] to obtain distinct field theories associated to the same quantum mechanical potential. For this purpose, we suggest different models related by the deformation procedure, which allows to write a constraint that ensures equality between the stability potentials of the models. The simplest deformation function which satisfies such constraint permits to study several examples having the feature indicated above. The procedure adopted here works as follows: given a starting model, the deformation is applied to provide other models that present the same stability potentials of the first one. The starting model may have symmetric or asymmetric topological sectors, and in the case of asymmetric sectors, the deformation procedure gives rise to a new sector, with modified stability potential, besides the ones having the same stability potential, as we show below.

The examples studied in this paper highlight polynomial and hyperbolic potentials. These kinds of models are useful to investigate defect structures [26,27], braneworld scenarios with a single extra dimension of infinite extent [28], kink–antikink pairs production in collisions of particle-like states [9], successions of phase transitions in field theories [29], and can also be employed to explore aspects of integrability [30–32].

There are other motivations to study kinks, since they may be used to model one-dimensional structures that appear in magnetic materials at the nanometric scale. For instance, in [33] the authors have experimentally shown that the kinklike profile may change when it is investigated on constrained geometries. Also, in [34] it was experimentally proven that electric pulses may contribute to change the polarity of a domain defect. More recently, authors have studied kinklike structures in interaction with elastic waves, as in [35], where magnetic bubbles in a bismuth-substituted iron garnet was considered, and also in [36], where a theoretical investigation suggests how to understand the interaction of elastic waves with domain walls.

In this work we concentrate on the construction of kinklike models that engender the same stability potential. In the next section, we review some results about the deformation procedure in systems involving a single real scalar field [25]. In Section 3 one shows how to find deformation functions that give rise to distinct field theories having the same stability potential. In Section 4, we present illustrations obtained by applications of a deformation function to several models with polynomial and hyperbolic interactions. We finish the work adding our comments and conclusions in Section 5.

2. Generalities

Let us start briefly revising the deformation procedure introduced in Ref. [25] for real scalar fields in $(1,1)$ spacetime dimensions. We will use natural units, with $\hbar = 1 = c$, and shift the field, spacetime coordinates and all the parameters that identify the model to work with dimensionless variables. We take two models defined by the Lagrangian densities

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi) \tag{1a}$$

$$\mathcal{L}_d = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{1b}$$