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CP symmetry and thermal effects on Dirac bi-spinor spin–parity local correlations

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A B S T R A C T

Intrinsic quantum correlations supported by the $SU(2) \otimes SU(2)$ structure of the Dirac equation used to describe particle/antiparticle states, optical ion traps and bilayer graphene are investigated and connected to the description of local properties of Dirac bi-spinors. For quantum states driven by Dirac-like Hamiltonians, quantum entanglement and geometric discord between spin and parity degrees of freedom – sometimes mapped into equivalent low energy internal degrees of freedom – are obtained. Such *spin–parity* quantum correlations and the corresponding nonlocal intrinsic structures of bi-spinor fermionic states can be classified in order to relate quantum observables to the (non)local behavior of these correlations. It is shown that free particle mixed states do not violate the Clauser–Horne–Shimony–Holt inequality: the correlations in such mixed bi-spinors, although quantum, can be reproduced by a suitable local hidden variable model. Additionally, the effects due to a non-minimal coupling to a homogeneous magnetic field, and to the inclusion of thermal effects are evaluated, and quantum correlations of associated quantum mixtures and of the thermal states are all quantified. The above-mentioned correlation quantifiers are then used to measure the influence of CP transformations on *spin–parity* quantum correlations, and our results show that quantum entanglement is invariant under CP

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transformations, although the geometric discord is highly sensitive to the CP symmetry.

1. Introduction

Since the Einstein, Podolsky and Rosen [1] controversial conclusion that quantum mechanics was not complete and that some underlying (hidden) variable would be necessary for a complete physical description of reality, mathematical and physical implications of a *(local) hidden variable theory* related to the phenomenon of quantum entanglement have been extensively investigated [2–6]. The so-called hidden variable mechanism should support the interpretation of quantum mechanics as to account for the probabilistic features driven by some kind of inaccessible variable theory. A local hidden variable (LHV) theory adds the local realism as a requirement to rule out from the theory any kind of instantaneous non-causal measurement events.

According to the Bell's theorem [7], some sets of LHV's cannot reproduce the quantum measurement correlations predicted by quantum mechanics. The novel element introduced by Bell [7] was that quantum correlations are essentially nonlocal. From the perspective of quantum entanglement [8–11], it states that separated particles can instantaneously share common properties and respond to quantum measurements as if they were a single particle. This provides the setup for the analysis of the hidden variable scenario, leading for instance to Bell's inequalities [5,7,12]. In such a context, the Bell inequality derived by Clauser, Horne, Shimony and Holt [13] – the CHSH inequality – has been used as a tool for testing locality in quantum systems. Through several setups, for example, in the characterization of superconducting qubits [14], in the architecture of trapped atoms [15], in the manipulation of quantum information protocols [5,6], or even in connection with noncommutative effects [16], the CHSH inequality has been supported by measurements of observable quantities.

Bell's inequality has also been the ground for discussing quantum correlations in relativistic setups [17–23]. Bell type correlations were firstly contextualized in a relativistic framework for discuss the concept of a relativistic center-of-mass [17], and since then the relation between Bell's inequality and transformation properties of quantum correlations was intensively studied [18–23]. For instance, when a pair of spins with different momenta is subjected to a Lorentz boost, the degree of violation of Bell's inequality is degraded [17–19], although for suitable conditions an anomalous behavior with local maximum of CHSH correlation, was observed [21]. A complete covariant setup was described for states constructed as a pair of Dirac particles with a more complex behavior of CHSH spin–spin correlations under frame transformations [20]. This relativistic effects on non-local spin correlations is due to the action of Lorentz boosts in spin states through a momentum dependent rotation, the Wigner rotation [24], and is the basis of many results in the fruitful field of relativistic quantum information [25]. The behavior of spin–spin and spin–momentum entanglement under relativistic transformations has also been addressed [23,26–28] and parallel to the discussion of transformation properties of quantum correlations, the very definition of relativistic spin operator has also been addressed [22,29–34]. In this fundamental context, a spin density matrix exhibiting covariant observable properties can be constructed with properly defined solutions of the Dirac equation [34,35].

The question to be posed in this manuscript is concerned with the quantification of the correspondence between (non)locality and quantum correlation aspects of the intrinsic structure of spin-1/2 particles described by relativistic bi-spinors, namely the (interacting) Dirac equation solutions. Relativistic(-like) quantum systems has been preliminary considered under the perspective of quantum information theory [36–38] in a framework to test fundamental features of the relativistic quantum mechanics, in particular, those related to the computation of quantum correlations and their (covariant) transformation properties [37,38]. Supported by a $SU(2) \otimes SU(2)$ group structure [39,40], Dirac bi-spinors exhibit a complete entanglement profile driven by two internal degrees of freedom – the *spin* and the *intrinsic parity/chirality* [39,41] – which provides an overall classification of the informational content of such Dirac-like structures [39,40].