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Construction and applications of the manifestly gauge invariant expressions of the solutions of the cosmological perturbation theory

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ABSTRACT

After giving how to construct the gauge invariant perturbation variable in an arbitrarily high order and in the arbitrary background spacetime, we consider the manifestly gauge invariant theory of the cosmological perturbations in the long wavelength limit with the spatially flat homogeneous isotropic universe being the background spacetime. In the previous paper, the physical laws, such as the evolution equations, the constraint equations and the junction conditions become manifestly gauge invariant, by writing them in terms of only the single time background/scalar like objects defined by our previous paper. In the present paper, by extending our definition of the background/scalar like objects from the single time case to the many time case and by writing the solution of the physical law in the form where many time background/scalar like objects are vanishing, the solutions become the manifestly gauge invariant. We derive the formula changing the bases of the many time background/scalar like objects by which we can change the time slices for many times appearing in the solution including the initial time and the final time. We use this formula to our treatment of the evolution of the several slow rolling scalar fields using the τ function introduced by our previous paper. In the manifestly gauge invariant manner, we discuss the solution of the many step reheating, that is, the reheating with the arbitrarily many energy transfer steps. Using the energy density ρ as the evolution parameter, we discuss how well the junction model in which the energy transfers from the oscillatory scalar field fluid to the radiation fluid are described by the metric junctions approximates the reheating process described by the differential equations with the decay terms. By using the useful parametrization of the many

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step reheating, we manifestly prove that in the many step reheating system where any initial perturbation of each component does not become extremely large compared with the initial perturbations of the other components, compared with the effects of the prominent fluctuation generation processes the nearest to the present time the effects of all the fluctuations generated by the previous fluctuation generation processes become negligibly small. Therefore it can be concluded that in order to know the first/second order Bardeen parameters at the present time it is sufficient to calculate only the effects of the fluctuation of the scalar field fluid which becomes energetically dominant lastly (the curvaton mechanism) and the fluctuation of the decay parameter by which this scalar field fluid decays into the radiation (the modulated reheating mechanism).

1. Introduction

In the inflation paradigm [1,2], after being stretched by the inflationary expansion, the cosmological perturbations generated by the slow rolling scalar fields and causing the structure formation and the fluctuations of the cosmic microwave background radiation stay on the superhorizon scales until they enter into the horizon in the post Friedmann epoch [3–7]. Therefore we want to calculate the evolutions of the cosmological perturbations on superhorizon scales including nonlinear levels in the case where the background spacetime is the spatially flat homogeneous isotropic universe. It was established that the evolutions of the cosmological perturbations in the long wavelength limit are constructed from the derivatives with respect to the solution constants of the solutions of the corresponding exactly homogeneous universe [8,9]. This fact has been used and has been developed in the various situations, by the author of the present paper [10–14] and with the reinterpretations in the contexts of each authors' researches [15–17]. In the most general situation in which plural scalar fields and plural perfect fluids coexist, the solutions of the evolutions of the cosmological perturbations in the long wavelength limit based on the corresponding exactly homogeneous perturbations are manifestly presented in the paper [12] and the paper [13] in the linear level and in the nonlinear level, respectively. Such approach was pioneered by the paper [18] where in the linear perturbation the expression of the adiabatic growing mode was given in terms of the exactly homogeneous solution, whose existence reason was given in the paper [9] and where the authors tried to give the expressions of the other perturbation solutions in terms of the background variables using the fact that all the coefficients of the linear perturbation equation with vanishing wavenumber are written in terms of the background variables only.

The perturbations are the differences between the physical quantities of the background homogeneous isotropic universe and those of the perturbed real universe. How to connect the physical quantity of the perturbed universe with the spacetime point of the corresponding homogeneous universe in order to take the difference is called the gauge. The changes of the perturbations generated by changing the gauge are apparent, do not have any physical meanings. The physical perturbations must be described by the perturbation variables which are gauge invariant, that is, do not change by changing the gauge. The theories in which the physical laws are described by the gauge invariant perturbation variables only are called the gauge invariant perturbation theories. The gauge invariant theory of the cosmological perturbation was constructed in the papers [3,4,19,20] in the linear perturbation level, and in the papers [21,22] in the second perturbation level. In an arbitrarily higher order nonlinear perturbation level, the treatment of the cosmological perturbations on the superhorizon scales in the manifestly gauge invariant manner was attained in the papers [13,14]. In the paper [23], Nakamura discussed how to construct the gauge invariant perturbation variable in the arbitrarily high order and in the arbitrary background spacetime. The answer to the problem presented by Nakamura will be given by our present paper. By using the infinitesimal gauge transformation and the application of induction using the concept of "the background like object",