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Extension of the input–output relation for a Michelson interferometer to arbitrary coherent-state light sources: *Gravitational-wave detector and weak-value amplification*

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ABSTRACT

An extension of the input–output relation for a conventional Michelson interferometric gravitational-wave detector is carried out to treat an arbitrary coherent state for the injected optical beam. This extension is one of necessary researches toward the clarification of the relation between conventional gravitational-wave detectors and a simple model of a gravitational-wave detector inspired by weak-measurements in Nishizawa (2015). The derived input–output relation describes not only a conventional Michelson-interferometric gravitational-wave detector but also the situation of weak measurements. As a result, we may say that a conventional Michelson gravitational-wave detector already includes the essence of the weak-value amplification as the reduction of the quantum noise from the light source through the measurement at the dark port.

1. Introduction

Weak measurements and their weak-value amplifications have been currently discussed by many researchers since their proposal by Aharonov, Albert, and Vaidman in 1988 [1]. In particular, the weak-value amplification has been regarded as one of techniques that has been used in a variety

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of experimental settings to permit the precise measurement of small parameters [2]. This paper is motivated by these researches on the precise measurements in quantum theory.

As well-known, one of typical examples of precise measurements is the gravitational-wave detection. Recently, gravitational waves are directly observed by the Laser Interferometer Gravitational-wave Observatory (LIGO) [3] and the gravitational-wave astronomy has begun. To develop this gravitational-wave astronomy as a precise science, improvements of the detector sensitivity is necessary. So, it is important to continue the research and development of the science of gravitational-wave detectors together with the source sciences of gravitational waves. This paper is also based on such research activities.

Although some researchers already commented that the weak-value amplification might be applicable to gravitational-wave detectors, we have been discussed this issue, seriously. The idea of weak measurements also proposed a new view-point of quantum measurement theory together with an amplification effect. To discuss the application of this idea to gravitational-wave detectors not only leads us to a possibility of exploring a new idea of the gravitational-wave detection but also gives us a good opportunity to discuss what we are doing in conventional gravitational-wave detectors from a different view-point of quantum measurement theory. Therefore, it is worthwhile to discuss whether or not the idea in weak measurements is applicable to gravitational-wave detector from many points of view. In particular, the comparison with conventional gravitational-wave detectors is an important issue in such discussions.

A simple realization of the weak-value amplification is similar to the gravitational-wave detectors in many points. The base of the conventional gravitational-wave detectors is the Michelson interferometer. The arm lengths of this Michelson interferometer are tuned so that the one of the port of the interferometer becomes the "dark port" as we will explain in Section 2. Due to the propagation of gravitational waves, photons leak to the "dark port". The measurement of the photon number at the "dark port" corresponds to the post-selection in weak measurements. This setup is regarded as a measurement of the effective two-level system of the photon. For this reason, we have been concentrated on the researches on weak measurements for two-level systems [4–7]. In particular, a weak-value amplification in a shot-noise limited interferometer was discussed [5], since the shot-noise is one of important noise in gravitational-wave detectors.

Recently, Nishizawa [7] reported his arguments on the radiation-pressure noise in a weak-measurement inspired gravitational-wave detector. This radiation-pressure noise is also an important noise in gravitational-wave detectors. He also discussed "standard quantum limit", which is a kind of the sensitivity limit of the detector, and proposed an idea to break his standard quantum limit. Details of the detector model inspired by weak measurements in Refs. [5,7] will also be explained in Section 2. In this detector model, the optical short pulse beam is used to measure the mirror displacement due to gravitational waves, while the continuous monochromatic laser is used for the continuous measurement of the mirror displacement in conventional gravitational-wave detectors. This short-pulse injection is one of ideas in weak measurements proposed by Aharonov et al. [1] and the main difference between a model inspired by weak measurements in Refs. [7] and conventional gravitational-wave detectors. Furthermore, in Ref. [7], arguments are restricted to the situation where the mirror displacement is regarded as a constant in time, while we have to monitor the motion of the mirror displacement by the continuous laser in conventional gravitational-wave detectors. Due to this restriction, we cannot directly compare the results in Ref. [7] with those in conventional gravitational-wave detector and the meaning of "standard quantum limit" in Ref. [7] is not so clear.

To monitor the time-evolution of the mirror displacement is important in gravitational-wave detection, because it corresponds to the monitor of the time-evolution of gravitational waves. Expected gravitational-wave signals are in the frequency range from 10 Hz to 10 kHz. When we apply the detector model in Ref. [7], we may inject femto-second pulses into the interferometer and a sufficiently large number of pulses are used to measure 10 kHz signals. Since we want to continuously measure the time-evolution of gravitational-wave signal in the range 10 Hz–10 kHz, we have to evaluate the averaged data of many pulses, continuously. To accomplish this averaged measurement, the different treatment of the detector from Ref. [7] is required. In conventional gravitational-wave detectors, the response of the detector to gravitational waves is discussed through the input–output relation of the interferometer in the frequency domain in the range of frequencies of gravitational-waves [8]. Therefore, to compare the results with conventional gravitational-wave detectors, it is