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# Spectral singularities, threshold gain, and output intensity for a slab laser with mirrors

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ABSTRACT

We explore the consequences of the emergence of linear and nonlinear spectral singularities in TE modes of a homogeneous slab of active optical material that is placed between two mirrors. We use the results together with two basic postulates regarding the behavior of laser light emission to derive explicit expressions for the laser threshold condition and output intensity for these modes of the slab and discuss their physical implications. In particular, we reveal the details of the dependence of the threshold gain and output intensity on the position and properties of the mirrors and on the real part of the refractive index of the gain material.



## 1. Introduction

A slab of homogeneous gain material that is placed between a pair of parallel mirrors, as depicted in Fig. 1, provides a simple model for a laser. The mirrors act as the boundaries of a Fabry–Perot resonator that enhance the optical path of the wave inside the gain region, hence reducing the threshold gain. The laser oscillations start once the gain coefficient $g$ for the system exceeds its threshold value $g_0$. The produced laser light can then exit the system through the mirror with higher transmission (lower reflection) coefficient.

In principle regardless of the details and geometry of a laser, in our case the type of the gain material and the presence and properties of the mirrors, laser light emission involves the generation of outgoing coherent electromagnetic waves. Purely outgoing solutions of a wave equation define its resonances [1]. If such a solution is required not to decay and behave like a scattering solution, i.e., tend to a plane wave in spatial infinities, then its wavenumber and consequently its energy

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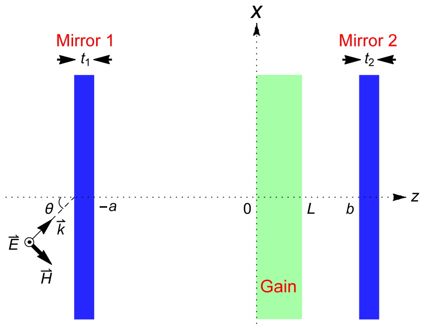


**Fig. 1.** (Color online) Schematic representation of a planar slab of thickness $L$ that is made out of a homogeneous gain material, and surrounded by mirrors.

must be real. Recalling that the imaginary part of the energy of a resonance determines its width, we can relate laser light emission to certain zero-width resonances. Ref. [2] identifies these with the mathematical notion of a spectral singularity [3–5]. A remarkable outcome of this identification is that the existence of a spectral singularity for an optical potential describing an active system coincides with the laser threshold condition for the system, i.e., $g = g_0$ for a homogeneous slab laser. An explicit demonstration of this result is provided in Ref. [6] for normally incident TE modes of a mirrorless slab laser and subsequently used as a computational scheme for the determination of the threshold gain for bilayer gain media [7,8] and media with spherical or cylindrical geometries that lase in their radial [9] or whispering gallery modes [10].

Another interesting application of spectral singularities is in the study of coherent perfect absorption (antilasing) [11–14]. A system serves as a coherent perfect absorber provided that the complex-conjugate of the associated optical potential has a spectral singularity [7]; in effect spectral singularities provide the basic mathematical tool for describing lasers and antilasers. For further discussion of physical aspects and applications of spectral singularities, see [15–26].

Another characteristic feature of lasing is its nonlinear nature [27]. Once the gain coefficient exceeds its threshold value and the laser oscillations begin, the propagation of purely outgoing waves inside the gain medium leads to a nonlinear response of the medium. This observation has motivated a generalization of the notion of spectral singularity to nonlinear wave equations [28–30]. Just above the threshold the nonlinearity can be treated as a first-order perturbation of the relevant Helmholtz equation. Refs. [31–33] show that if we identify this perturbation with a weak Kerr nonlinearity, then the condition for the emergence of a nonlinear spectral singularity yields an expression for the laser output intensity $I$ that is linear in the gain coefficient $g$. More specifically,

$$I = \left(\frac{g - g_0}{\sigma g_0}\right)\widehat{I}, \tag{1}$$

where $\sigma$ is the Kerr coefficient, and $\widehat{I}$ is a function of the geometry and other parameters of the system. The linear dependence of $I$ on $g$ is one of the basic results of laser physics. Here it follows from the purely mathematical condition of the existence of a nonlinear spectral singularity [28].

In the present article, we explore the linear and nonlinear spectral singularities in the TE modes of a homogeneous slab of gain material that is placed between a pair of parallel mirrors. Our aim is to determine the explicit form of the threshold gain $g_0$ and the slope $\widehat{I}$ of the intensity without having to rely on any physical arguments. More precisely, we offer a derivation of the laser threshold condition and the linear-dependence of the intensity on the gain coefficient using the following two basic postulates:

Postulate 1: The emitted laser light is purely outgoing.
Postulate 2: The interaction of the wave with the slab is described in terms of a weak Kerr nonlinearity.