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Weak values in collision theory

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A R T I C L E I N F O

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A B S T R A C T

Weak measurements have an increasing number of applications in contemporary quantum mechanics. They were originally described as a weak interaction that slightly entangled the translational degrees of freedom of a particle to its spin, yielding surprising results after post-selection. That description often ignores the kinetic energy of the particle and its movement in three dimensions. Here, we include these elements and re-obtain the weak values within the context of collision theory by two different approaches, and prove that the results are compatible with each other and with the results from the traditional approach. To provide a more complete description, we generalize weak values into weak tensors and use them to provide a more realistic description of the Stern–Gerlach apparatus.

1. Introduction

New insight in science sometimes is gained when two different fields of knowledge are recognized for the similarities they bear. Weak measurements, first proposed in the 1980s by Aharonov et al. [1], and the much older formalism of collision theory are two such theories containing striking parallels. In this article, we seek to survey these similarities.

The concept of weak measurement, despite being quite more recent, draws from sources as old as quantum mechanics itself, such as von Neumann's formalism for the measurement. In his pioneering textbook, von Neumann [2] dealt with the measurement process in quantum mechanics as a unitary evolution governed by an interaction Hamiltonian H_{int} between the main system and the measurement apparatus. After this process, during which the two systems end up correlated,

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occurs the reduction of the total state. At this point the terminology presents a series of variations: according to [3], the first step, when the unitary evolution occurs, is called pre-measurement, while the collapse of the wave function is called measurement; Peres [4], on the other hand, calls this global process an intervention, divided in an interaction portion called measurement, and the collapse which constitutes the output. This work by von Neumann was employed as a starting point by many approaches to quantum mechanics, from Everett's relative state formulation [5–7] to the decoherence theory [8–12] and pointer states [13–16], up to analyses of the influence of the intensity of the interaction on the probabilities associated with each result [17–19] and, finally, to weak values and measurements [1,20–22].

The concept of weak value emerged from the analysis made by Aharonov et al. [23] of an ensemble both pre- and post-selected, in an attempt to construct a new time-symmetric quantum theory. Even though the concept of post-selection can initially sound uncanny [24–26], we must highlight that we are not dealing with an isolated system, but actually with two interacting Hilbert spaces, only one of which suffers post-selection — a clear exposition of this is given by Duck et al. [22]. Between the pre- and the post-selection, the intensity of the interaction H_{int} is made weak enough so the higher-order terms in the power expansion of the time evolution operator can be discarded, causing some "surprising quantum effects" [20]. Namely, the weak value can surpass the limits defined by the eigenvalue spectrum of the observable [1,22] and, as the weak values result from a weak interaction, the state of the measured system remains practically unchanged — that is, even though we are talking about "measurement", there is no collapse of the wave function.

In short, determining the weak value takes three measurements: (1) the preparation of the initial state – or pre-selection – which can be performed using a regular measurement; (2) the weak measurement proper (a weak interaction that barely perturbs the state of the system); and (3) the regular projective measurement, performed in the post-selection stage. A collision experiment [27–29] is very similar to a weak measurement: there is an initial preparation of the state that will be collided, the parts of the system that will collide approach and interact during a finite time, and again depart, until the point when, asymptotically, cease to interact, and, finally, are detected by an apparatus that performs the final measurement, or post-selection. The differential cross section, found by means of a collisional experiment, is given in terms of the scattering matrix [28,30,31].

A relationship between the scattering matrix and the weak values has been established in [32]. However, the authors treated the scattering matrix as a response function of the system, exemplifying their set-up by the construction of an effective scattering matrix applied to an optical experiment using bi-refringent crystals.

In this work, we propose to establish a more specific parallel with the formal collision theory, and apply it, aiming to create the possibility of actual experimental set-ups with realistic collision, as in the case of ultracold atomic collisions [33]. Other controversial questions about the weak value [34–38] will not be dealt with here and will not be necessary for what we propose to prove here. As recent reviews on the subject suggest [38–40], the concept of weak value has become less abstract, and now can be built from classical statistics [41]. Worth of mention are the proposal to use weak measurements to protect the state with finite-time measurements [19], and important experimental implementations, for example, in optical interferometry to analyze the classical two-slit experiment [42], and in condensed matter [43–45].

In Sections 2 and 3, we will revise the concepts of weak of measurements and collision theory, respectively, which we will apply concomitantly in Section 4. Conclusions are presented in Section 5.

2. Review of weak values

Following the original article about weak measurements by Aharonov et al. [1,21] and the alternative description by I. M. Duck et al. [22], in this section we will consider an interaction that entangles two quantum subsystems, one in a continuous Hilbert space and the other in a discrete Hilbert space. We will treat these two subsystems as a position (represented by the state vector $|\Phi\rangle$) and a spin (represented by$|\chi\rangle$) of a particle, just like in the Stern–Gerlach experiment. Initially, we will assume they are not entangled to each other:

$$|\Psi_{\mathrm{i}}\rangle = |\Phi_{\mathrm{i}}\rangle \otimes |\chi_{\mathrm{i}}\rangle \, , \tag{1}$$