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Super periodic potential

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HIGHLIGHTS

- Concept of Super Periodic (SPP) potential of arbitrary order is introduced.
- Scattering coefficients for arbitrary SPP are calculated in closed form.
- Symmetric Fractal potential is derived as a special case of SPP.
- Generalized Cantor and Smith–Volterra–Cantor (SVC) potentials are studied as special cases of SPP.
- Tunnelling amplitudes for fractal, Cantor and SVC potentials are derived.

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ABSTRACT

In this paper we introduce the concept of super periodic potential (SPP) of arbitrary order $n, n \in I^+$, in one dimension. General theory of wave propagation through SPP of order n is presented and the reflection and transmission coefficients are derived in their closed analytical form by transfer matrix formulation. We present scattering features of super periodic rectangular potential and super periodic delta potential as special cases of SPP. It is found that the symmetric self-similarity is the special case of super periodicity. Thus by identifying a symmetric fractal potential as special cases of SPP, one can obtain the tunnelling amplitude for a particle from such fractal potential. By using the formalism of SPP we obtain the close form expression of tunnelling amplitude of a particle for general Cantor and Smith–Volterra–Cantor potentials.

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1. Introduction

There is always a recurring interest, both theoretical and experimental, in the study of one dimensional scattering by finite periodic chains of non-overlapping barriers or wells. This problem has its own significance because it exhibits important features of quantum mechanics: tunnelling and interference [1,2] which serves practical motivations for the physics of lattices and superlattices in solid state physics and electronic devices [3], optical multiquantum well systems, and theory of X-ray reflection from amorphous superlattices [4]. The historic paper of Kronig and Penney (1930) , describes motion of an electron in an infinite (or half-infinite) one-dimensional periodic chain of delta-potentials [5], and has served as the most important tool in explaining various interesting physical properties of real materials. Similar problem with a finite number of scattering centres has also been studied. Pshenichnov (1962) has investigated the scattering problem on a periodic finite chain of identical potentials using the WKB approximation and has shown the existence of resonance effect in the transmission coefficient [6]. Reading and Siege(1972) have considered particle scattering from a finite chain of delta-potentials of arbitrary strengths and positions using momentum representation method [7]. The scattering problem for a finite chain of N equally spaced symmetric potentials has been studied by many groups in twentieth century by various methods and formalisms [8,9]. Scattering from finite and infinite repeated structures to see the band-structure characteristics of periodic potentials along with resonances are also observed [1,10–12].

However, the realization of scattering and resonances from a super periodic potential with multiple periodicities which is the most appropriate and generalized representation of superlattices are still phenomenological and there is no concrete theoretical analysis present in literature, to the best of our knowledge. Superlattices are in general a periodic structure of layers of two or more materials, typically having the thickness of each layer of several nano-metres. Many artificially created superlattices have been proposed with various solid state systems with utmost two or three patterns of repetitions and some are also experimentally studied [13]. It will be extremely helpful to have the generalized version of scattering matrices and coefficients for any multiple patterns of repetition to make more and more advanced superlattices. In parallel, some super-periodic structures are also invented in optics to serve practical purposes in modern science and technologies [14,15] with two or three periodicities, but without having any analytical generalized form of the scattering matrices. No attempt has been made yet for constructing the generalized form of scattering matrices for a super-periodic potential with arbitrary number periodicities. Of course this problem is quite challenging because of its extremely complicated structure and analytical lengthiness. In this work we not only develop the most generalized structure of super-periodic potentials but also explicitly construct the corresponding transfer matrices and scattering coefficients in the closed forms.

Our aim in this article is to present the exact scattering solutions and resonances for such an array of quantum mechanical potentials which is distributed in a finite or infinite space with a super-periodic manner. This super periodic potential of order 'n' is described as follows: consider a 'unit cell potential' of width $2a$ periodically repeats N_1 times with the constant distance between two successive cells as c_1, which is its periodic distance. This periodic system 'as a whole' repeats N_2 times with periodic distance c_2. This new system again wholly repeats N_3 times with periodic distance c_3. This process of periodic repetition, of a new potential system each time, continues up to an arbitrary number of times N_n with the periodic distance c_n, where n is positive integer. The closed form expression of the scattering coefficients (reflection and transmission) for such a generalized super periodic distribution of several sets of potentials is constructed in this work. We treat the problem of order n in terms of the transfer matrix for each unit cell potential. Specific examples of super periodic potential are discussed when unit cell potential is delta and rectangular potential.

It is an extraordinary fact to see that a family of symmetric fractal potentials is the special cases of super periodic potential. This shows that the concept of super periodicity is more general than the periodicity and being the super set of a family of symmetric potential broaden its scope to great length. Thus identifying a fractal potential as super periodic potential completes the theory of tunnelling amplitude from such potential in analytical form. There have been extensive works on the tunnelling problem of potentials which has fractal distributions [16–23]. Much of these works have relied on the numerical multiplication of the transfer matrices as well as their analytical properties. We identify