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A duality web in condensed matter systems

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ABSTRACT

We study various dualities in condensed matter systems. The dualities in three dimensions can be derived from a conjecture of a duality between a Dirac fermion theory and an interacting scalar field theory at a Wilson–Fisher fixed point and zero temperature in three dimensions. We show that the dualities are not affected by non-trivial holonomy, use a mean-field method to study the dualities, and discuss the dualities at a finite temperature. Finally, we combine a bulk theory, which is an Abelian p-form theory with a theta term in $2p+2$ dimensions, and a boundary theory, which is a $2p+1$ dimensional theory, to discuss constraints and difficulties of a $2p+1$ dimensional duality web.

1. Introduction

Duality can exhibit non-trivial equivalence between two different theories. A generic study of a duality web appears in string theory and supergravity theory [1]. String theory and supergravity theory show a beautiful duality web to unify different fundamental theories through T-duality and S-duality [2]. S-duality can exchange a weak coupling constant and a strong coupling constant to demonstrate equivalence between a weakly coupled effective field theory and a strongly coupled effective field theory. Hence, S-duality can show non-perturbative physics through a perturbative study. The electric–magnetic duality in an Abelian p-form gauge theory in $2p+2$ dimensions is studied in [3,4].

A duality web in condensed matter systems is found in [5] for a generic background at an infrared (IR) limit and zero temperature in three dimensions by considering a $Spin_c$ manifold. The motivation of introducing a $Spin_c$ manifold comes from an inconsistent dualization of a conjecture [5], which is equivalence between a Dirac fermion and an interacting scalar field theory, in a *Spin* manifold. Because we expect that the conjecture should be correct for a generic background, the dualization

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of the conjecture should not show inequivalent dependence of a choice of a spin structure [5]. Therefore, a generalization from a *Spin* manifold to a $Spin_c$ manifold is useful for using a larger structure to study three dimensional condensed matter systems. The related study of the duality web for a flat background is in [6], the duality web at a finite temperature is in [7], a duality between a four dimensional topological insulator and a four dimensional topological superconductor is in [8], dualities for an *SO* gauge group are in [9,10] and dualities for a *USp* gauge group are in [10], a lattice study in a particle–vortex duality is in [11], and a derivation of dualities by promoting background fields to dynamical fields is in [12]. Other useful studies of a mirror symmetry for a supersymmetric defect is in [13], a half-filled Landau level is in [14], a bosonization is in [15] and non-supersymmetric dualities are in [16].

An effective field theory in the conjecture is an Abelian Chern–Simons theory, which is topological quantum field theory, at an IR limit in three dimensions. The Abelian Chern–Simons theory with a one-half coefficient can be generated from a one-loop effective potential of a massless Dirac fermion with a regularization [17] in three dimensions [18]. The massless Dirac fermion does not have parity violation in three dimensions at classical level, but the parity anomaly appears at quantum level [19]. The parity anomaly with gravitation is also studied in [20] and the parity anomaly in an unorientable manifold is discussed in [21]. A discussion of a canonical quantization of the Abelian Chern–Simons theory is in [22] and a holomorphic view of three dimensional topological quantum field theory is in [23]. More useful studies of fermion path integrals are in [24] and topological phases are in [24,25].

The three dimensional duality web can also be understood from modular transformations or $SL(2)$ transformations in a four dimensional Abelian gauge theory with a theta term in a manifold with a boundary [5] at an IR limit. When we consider the four dimensional theory or the bulk theory and the three dimensional Dirac fermion theory or the boundary theory simultaneously, we can find that the four dimensional dynamical gauge field becomes a background gauge field by taking an IR limit, which leads a weak four dimensional coupling constant. The IR limit also leads a strong three dimensional coupling constant. Three dimensional dynamical gauge potential is still dynamical at the IR limit and the three dimensional gauge potential is also coupled to a scalar field. The scalar field theory is another boundary theory, in which a gauge potential is dynamical. A combination of the bulk theory and the boundary theory can be studied self-consistently. The study also demonstrates a more reliable result. If we only consider three dimensions or a boundary theory at the IR limit, the duality web needs to begin from the conjecture. Thus, all results depend on whether the conjecture is correct. Form the combination of the bulk theory and the boundary theory, we can obtain alternative evidences for the conjecture. A demonstration of a three dimensional particle–vortex duality from the four dimensional Abelian gauge theory with a theta term is in [26].

In this paper, we discuss various dualities in condensed matter systems. We begin from the conjecture to show other dualities without losing holonomy. The dualization is to integrate gauge fields out, which is similar to solving $da = db$, where a and b are gauge fields. A general solution of $da = db$ is $a = b + df$ with $d^2 f = 0$ when a and b satisfy a Dirac quantization condition, which is $\int da = \int db = 2\pi$. When we set $f = 0$ or zero holonomy due to a gauge symmetry of a physical system, all fields of the physical system should be dynamical. If some fields are just background fields, the holonomy may deform the background fields after we do a dualization. Thus, the three dimensional duality web may be modified by the holonomy. However, we find that non-trivial holonomy can be absorbed by a field redefinition of a scalar field when a boundary term is absent [7]. Thus, the non-trivial holonomy does not modify the three dimensional duality web when a boundary term is ignored and all gauge fields satisfy a Dirac quantization condition or fields are globally defined.

We also use a mean-field method to study the three dimensional duality web. The study of the mean-field method also begins from the conjecture and use the order parameter $\bar{\psi}\psi$. Our study shows that the order parameter in a Dirac fermion theory can dual to a bosonic mass term in a scalar field theory [7]. Thus, it is interesting to find an operator correspondence between the fermion theory and the boson theory.

We also propose inclusion of a finite temperature in the three dimensional duality web. One difficulty of the duality web at a finite temperature is that a gauge invariant effective field theory from that a Dirac fermion theory needs to do resummation to all orders. The duality web at zero