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Dynamical transitions in a modulated Landau–Zener model with finite driving fields

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HIGHLIGHTS

- The rigorous dynamics of a modulated Landau–Zener model is investigated.
- Nonadiabatic evolution of the model could realize complete population transfer.
- Imperfect effects caused by both the amplitude and phase damping are estimated.

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ABSTRACT

We investigate a special time-dependent quantum model which assumes the Landau–Zener driving form but with an overall modulation of the intensity of the pulsing field. We demonstrate that the dynamics of the system, including the two-level case as well as its multi-level extension, is exactly solvable analytically. Differing from the original Landau–Zener model, the nonadiabatic effect of the evolution in the present driving process does not destroy the desired population transfer. As the sweep protocol employs only the finite driving fields which tend to zero asymptotically, the cutoff error due to the truncation of the driving pulse to the finite time interval turns out to be negligibly small. Furthermore, we investigate the noise effect on the driving protocol due to the dissipation of the surrounding environment. The losses of the fidelity in the protocol caused by both the phase damping process and the random spin flip noise are estimated by solving numerically the corresponding master equations within the Markovian regime.

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1. Introduction

Exactly solvable time-dependent quantum system attracts increasing interest owing to its role in the design for quantum control. In particular, to model dynamical processes or target quantum states for atomic and molecular systems [1,2], nonadiabatic transitions induced by time-varying external fields are often involved and the theoretical proposal of the driving protocol with desired dynamics is generally a prerequisite to accomplish the corresponding quantum tasks [3–13].

Landau–Zener (LZ) model [3,4] and its analogs, represented by the Hamiltonian below, are the most frequently exploited proposals in the driving protocol

$$H(t) = \Omega_x(t)J_x + \Omega_z(t)J_z. \tag{1}$$

Here $J_{x,z}$ denote the angular-momentum operators and $\Omega_{x,z}(t)$ account for two components of the driving field along the x and z axes, respectively. Owing to the explicit time dependency of $H(t)$, the general solution to this kind of systems is highly nontrivial even for the simplest two-level case, i.e., with the azimuthal quantum number $j = \frac{1}{2}$. For the standard LZ sweep with Ω_x being constant and $\Omega_z(t)$ varying linearly with time, the very two-level model is exactly solvable and the transition probability induced by the evolution over $t \in (-\infty, \infty)$ is known well as the LZ formula [3,4]. Notably, the LZ model has a wide range of applications in physics as well as in chemistry, including the LZ interferometry [14–17], the transfer of charge [18], chemical reactions [19,20], controllable manipulation of qubit and qutrit systems [21–24], and so on.

The so-called counter-diabatic protocol [5,6] (also named as the transitionless protocol [7] or shortcuts to adiabaticity [8]) has been proposed to generate exact dynamical evolution which aims at the adiabatic eigenstates, e.g., of a given Hamiltonian of form (1). Typically, this kind of protocols exploit a reverse-engineering strategy through introducing an auxiliary counter-diabatic driving term [e.g., an extra time-varying field along the y-axis which cancels out the nonadiabatic effect of $H(t)$] to ensure the desired evolution. We would also like to mention another reverse-engineering algorithm proposed in Ref. [9], where a parametric connection is established between the evolution operator and the control field of the Hamiltonian. In comparison, while the latter method is able to generate the LZ-type protocol with two driving components formed of Eq. (1), its applications were restricted to the two-level systems [9–11].

Except for the models constructed through the mentioned reverse-engineering methods, analytically exactly solvable time-dependent quantum systems are relatively rare and known examples are mostly concentrated on the two-level system, for example, the Rosen–Zener [25], Allen–Eberly [26], Demkov–Kunike [27,28], and Bambini–Berman [29] models. In a recent work, a tangent-pulse driven model has been proposed [30] which is shown to be analytically solvable not only for the two-level case but also for the multi-level extension. The nonadiabatic dynamics generated by the model itself can serve as a desirable protocol for the population transfer without the need of any auxiliary fields. While the ideal design assumes an infinite chirping field, it is demonstrated that for an imperfect scanning process with truncation, the cutoff error caused to the population transfer could be suppressed to the infinity through enhancing the scanning rate of the protocol.

In this paper we propose a modulated LZ model and explore the generated dynamics for quantum control. In particular, we demonstrate that the model offers an alternative protocol for the nonadiabatic population transfer which retains the advantages previously displayed in the tangent-pulse driven model: the nonadiabatic evolution can realize complete population transfer and no auxiliary field is required; the model is genuinely solvable which can be extended to the multi-level system. Furthermore, since the present protocol employs only the fields of finite intensity, it avoids the nonrealistic design of infinite driving assumed in the original LZ model and other analogous schemes. Meanwhile, the cutoff error in the protocol due to the truncation of the scanning pulse to the finite time interval is shown to be negligibly small. To evaluate further the feasibility of the scheme in the real systems, we investigate the noise effect of the protocol under dissipation. We solve numerically the master equations associated with the dephasing process and the random spin flip process within the Markovian regime. The loss of the fidelity caused by the detrimental influence of the noise is estimated.