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# Potential landscape and flux field theory for turbulence and nonequilibrium fluid systems

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## HIGHLIGHTS

- The notion of nonequilibrium trinity based on detailed balance breaking is proposed.
- Nonequilibrium trinity captures the nonequilibrium irreversible nature of turbulence.
- Nonequilibrium trinity leads to the potential-flux form of stochastic fluid dynamics.
- Energy flux in turbulence energy cascade is connected to nonequilibrium trinity.
- The four-fifths law of turbulence is a manifestation of nonequilibrium trinity.

## ARTICLE INFO

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## ABSTRACT

Turbulence is a paradigm for far-from-equilibrium systems without time reversal symmetry. To capture the nonequilibrium irreversible nature of turbulence and investigate its implications, we develop a potential landscape and flux field theory for turbulent flow and more general nonequilibrium fluid systems governed by stochastic Navier–Stokes equations. We find that equilibrium fluid systems with time reversibility are characterized by a detailed balance constraint that quantifies the detailed balance condition. In nonequilibrium fluid systems with nonequilibrium steady states, detailed balance breaking leads directly to a pair of interconnected consequences, namely, the non-Gaussian potential landscape and the irreversible probability flux, forming a 'nonequilibrium trinity'. The nonequilibrium trinity characterizes the nonequilibrium irreversible essence of fluid systems with intrinsic time irreversibility

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and is manifested in various aspects of these systems. The nonequilibrium stochastic dynamics of fluid systems including turbulence with detailed balance breaking is shown to be driven by both the non-Gaussian potential landscape gradient and the irreversible probability flux, together with the reversible convective force and the stochastic stirring force. We reveal an underlying connection of the energy flux essential for turbulence energy cascade to the irreversible probability flux and the non-Gaussian potential landscape generated by detailed balance breaking. Using the energy flux as a center of connection, we demonstrate that the four-fifths law in fully developed turbulence is a consequence and reflection of the nonequilibrium trinity. We also show how the nonequilibrium trinity can affect the scaling laws in turbulence.



## 1. Introduction

The true nature of turbulence remains elusive despite more than a hundred years of devotion of countless geniuses since the pioneering work of Osborne Reynolds who investigated experimentally the transition from laminar to turbulent flow [1]. The energy cascade picture in turbulent flows proposed by Richardson [2] is arguably the most important physical picture of turbulence, in which energy flows from the large scale where energy is injected, transferred through the intermediate scales by the nonlinear convective force, down to the small scale where energy is dissipated by molecular viscosity. The modern viewpoint of turbulence started from Kolmogorov's ground-breaking theory of scaling laws [3,4] based on the hypotheses of universality and self-similarity [5,6] to quantify the energy cascade process. Later experimental observations demonstrated that self-similarity of the energy cascade in turbulence is broken due to the intermittency phenomena [7], which has inspired intensive investigations on this subject [8,9]. Now the modern turbulence research has developed into a vast field with a variety of branches.

As is well known, turbulence is a far-from-equilibrium phenomenon without time reversal symmetry [10,11]. The nonequilibrium irreversible character of turbulence plays an important role in various facets of the turbulence phenomenon. In particular, the energy cascade process, with a directional flow of energy through scales, is a manifestation and reflection of the nonequilibrium irreversible nature of turbulence. It is therefore natural to approach the turbulence problem from the perspective of nonequilibrium statistical mechanics [12–16]. The statistical properties of turbulence in connection with the deviation from equilibrium has been investigated from the angles of fluctuation–dissipation theorem (FDT) [17–19], fluctuation theorem [20–23], large deviation theory [24–26], and time asymmetry in Lagrangian statistics [27–31] among others.

However, a precise characterization of the nonequilibrium steady state with intrinsic time irreversibility for turbulent fluids governed by stochastic Navier–Stokes equations, based on the concept of detailed balance and its violation (i.e., detailed balance breaking) in stochastic dynamical systems [32–34], is still lacking to the best knowledge of the authors. In this respect the work on time asymmetry in Lagrangian statistics is relevant, which studied the manifestation of detailed balance breaking in the motion of fluid particles in the turbulent flow. The objective of the present work is to develop theoretically a systematic characterization of the intrinsic nonequilibrium irreversible nature of turbulent flow as a whole and investigate its manifestation in some major aspects of the turbulence phenomenon, within the framework of the potential landscape and flux field theory.

The potential landscape and flux field theory, a generalization of the potential landscape and flux framework to spatially extended systems (fields), is a theoretical framework that belongs to the larger field of stochastic approaches to nonequilibrium statistical mechanics. It is particularly suited for the study of the global dynamics and nonequilibrium thermodynamics of stochastic field systems governed by the Langevin and Fokker–Planck field equations [35–37]. The potential landscape and flux framework, which has its historical origin in the energy landscape theory in protein folding dynamics,