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Determination of many-electron basis functions for a quantum Hall ground state using Schur polynomials

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ABSTRACT

A method for determining the ground state of a planar interacting many-electron system in a magnetic field perpendicular to the plane is described. The ground state wave-function is expressed as a linear combination of a set of basis functions. Given only the flux and the number of electrons describing an incompressible state, we use the combinatorics of partitioning the flux among the electrons to derive the basis wave-functions as linear combinations of Schur polynomials. The procedure ensures that the basis wave-functions form representations of the angular momentum algebra. We exemplify the method by deriving the basis functions for the $\frac{5}{2}$ quantum Hall state with a few particles. We find that one of the basis functions is precisely the Moore–Read Pfaffian wave function.

1. Introduction

Fractional quantum Hall effect at the filling factor $\frac{5}{2}$ is the first example in condensed matter systems wherein quasi-particles are predicted to obey a non-Abelian braiding statistics [1,2]. At this filling, the second Landau level of spin-up electrons is half-filled. Moore and Read (MR) proposed a ground-state wave function [3] for this state with an effective filling factor $\nu^* = 1/2$ which can be

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interpreted as a chiral p-wave superfluid state of composite fermions [4] formed as a bound state of an electron and two flux quanta, each quantum being $\phi_0 = hc/e$. This fractional quantum Hall state is also popularly known as the "Pfaffian state" as the real-space representation of the chiral p-wave superfluid wave-function is a Pfaffian of a certain antisymmetric matrix depending on the relative positions of the particles. The edge of this state carries a neutral mode of Majorana fermions [5]. It occurs for a flux $\Phi = 2N - 3$ in units of ϕ_0 in a system of N electrons moving on the surface of a sphere [6] of radius $\sqrt{\Phi/2}$. On a disk, the orbitals occupied by N electrons have an angular momentum less than or equal to Φ. The quasi-holes of the MR state obey a non-Abelian braiding statistics [1,2] as the ground state of $2m$ quasi-holes is 2^{m-1}-fold degenerate [7]. This wave-function is the exact ground state wave-function for a model three-body pseudo potential [1]. The MR wave-function has also been shown to be equivalent to the $\mathbf{Z}_2$ parafermion [8] wave-function. Jack polynomials characterized by occupation number configuration have been introduced [9] for the MR states. It naturally implements a squeezing rule in the configuration. However, no better alternative wave-function for the $\nu^* = 1/2$ state has been derived from this type of configuration so far. A better wave-function is indeed necessary for understanding the ground state due for the Coulomb interaction.

In this article we propose a method to determine the basis wave-functions for a fractional quantum Hall state of N electrons, with the maximum number of filled single-particle orbitals (same as the total number of flux quanta Φ). The ground state of the system is a suitable linear combination of these wave-functions. While the procedure is valid for arbitrary Φ and N, we consider, by way of example, the "Pfaffian flux-shift" $\Phi = 2N - 3$, for which the MR wave-function [3] has been proposed as the ground state wave-function of the system. Numerical exact diagonalization studies [10], however, indicate that the MR wave-function is not the veritable ground state wave-function for the Coulomb interaction. The basis wave-functions obtained using our prescription should provide an accurate ground state.

Our method, being combinatorial in nature, has the advantage of being conceptually simple, if computationally demanding. The assumptions made are few and most natural, rendering the method very general. We obtain the basis wave-functions solely from the knowledge of the integral flux and the number of electrons.

2. Construction

Let us consider a collection of N electrons on the complex plane $\mathbf{C}$, at positions $\{z_i | z_i \in \mathbf{C}, i = 1, \ldots, N\}$ in a magnetic field with a given total integral flux Φ in units of the flux quantum ϕ_0. In other words, Φ is a given, arbitrary, positive integer. The wave-function of this many-particle system is sought in the form $\Psi(z_1, z_2, \ldots, z_N) e^{-\frac{1}{4}\sum_k |z_k|^2}$, where Ψ is a polynomial in the coordinate ring $\mathbf{C}[z_1, z_2, \ldots, z_N]$. We assume that the physical quantities derived using this wave function remain unaltered as the electrons are shuffled. Assuming further that Φ is distributed as flux lines between different pairs of electrons formed for a single electron, so that every unit of flux is divided between two electrons of the pair in two moieties, and every pair of electron feels at least one unit of flux, the total angular momentum of the collection is [6]

$$L = N\Phi/2. \tag{1}$$

Requiring the wave-function to furnish a representation of the angular momentum algebra implies that the polynomial Ψ is homogeneous of degree L. Each electron feels this flux. The maximal index of any z_i in Ψ is thus Φ. Let us define $z_{ij} = z_i - z_j$. The number of such variables is

$$E = \binom{N}{2}, \tag{2}$$

the number of ways N objects can be paired. Since two electrons are not allowed to be at the same position, the wave-function is supposed to vanish at $z_{ij} = 0$, for every pair of i and j. This is ensured by assuming that the polynomial Ψ is antisymmetric under the exchange of z_i and z_j. Without any loss of generality we assume it to be of the form

$$\Psi(z_1, \ldots, z_N) = \Delta(z_1, \ldots, z_N) S^{(N)}(z_1, \ldots, z_N), \tag{3}$$