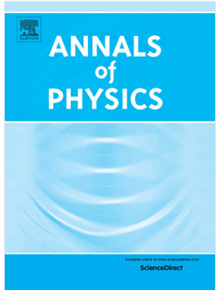

Spatial evolution of quantum mechanical states

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Spatial evolution of quantum mechanical states

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The time-dependent Schrödinger equation is solved traditionally as an initial-time value problem, where its solution is obtained by the action of the unitary time-evolution propagator on the quantum state that is known at all spatial locations but only at t=0. We generalize this approach by examining the spatial evolution from a state that is, by contrast, known at all times t, but only at one specific location. The corresponding spatial-evolution propagator turns out to be pseudo-unitary. In contrast to the real energies that govern the usual (unitary) time evolution, the spatial evolution can therefore require complex phases associated with dynamically relevant solutions that grow exponentially. By introducing a generalized scalar product, for which the spatial generator is Hermitian, one can show that the temporal integral over the probability current density is spatially conserved, in full analogy to the usual norm of the state, which is temporally conserved. As an application of the spatial propagation formalism, we introduce a spatial backtracking technique that permits us to reconstruct any quantum information about an atom from the ionization data measured at a detector outside the interaction region.