Annals of Physics 388 (2018) 100–113

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Squeezed coherent states of motion for ions confined in quadrupole and octupole ion traps

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ARTICLE INFO

Article history:
Received 9 May 2017
Accepted 3 November 2017
Available online 11 November 2017

Keywords:
Trapped ion
Squeezed coherent states
Anharmonic electric potential
Time-dependent variational principle
Octupole combined ion trap

ABSTRACT

Quasiclassical dynamics of trapped ions is characterized by applying the time dependent variational principle (TDVP) on coherent state orbits, in case of quadrupole and octupole combined (Paul and Penning) or radiofrequency (RF) traps. A dequantization algorithm is proposed, by which the classical Hamilton (energy) function associated to the system results as the expectation value of the quantum Hamiltonian on squeezed coherent states. We develop such method and particularize the quantum Hamiltonian for both combined and RF nonlinear traps, that exhibit axial symmetry. We also build the classical Hamiltonian functions for the particular traps we considered, and find the classical equations of motion.

1. Introduction

Recent advances in quantum optics [1] enable trapping of single particles or atoms [2,3], while progress in quantum engineering techniques allows preparing these particles in well-defined quantum states [4–6]. Quantum engineering using ion traps offers the possibility to prepare stable quantum states by precise control of the interactions between a quantum system (trapped ions) and the environment [7–9], while investigation of nonclassical states of spin systems coupled to a harmonic oscillator offers the possibility to investigate fundamental quantum phenomena, such as the mechanisms responsible for decoherence and the quantum–classical transition [10,11].

The quantum time-dependent harmonic oscillator has been intensively used to describe the dynamics of many physical systems [12–16]. Quantum dynamics of harmonic oscillators is obtained by the use of the so-called *Perelomov's generalized coherent states* [17,18] of the Lie algebra associated to the SU(1,1) group [19]. Quantum dynamics in a 3D RF ion trap characterized by a uniform magnetic

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https://doi.org/10.1016/j.aop.2017.11.004

field and a time-dependent quadrupole electric potential, can be reduced to the solution of the time-dependent quantum oscillator equations, obtained by separating the axial and radial motion from the Schrödinger equation [20]. In addition, the coherent state approach leads to quantum solutions that are explicitly constructed as functions of the classical trajectories on the phase space [18]. The properties of a RF (Paul)-trap with a super-imposed magnetic field (combined trap) are presented in [21], and it was shown that the regions of stability are significantly larger than those for a Paul trap for both positive and negative charged ions. Ion dynamics in a radiofrequency (RF) octupole trap was described in [22,23], demonstrating confinement of ions in a RF anharmonic electric potential and characterizing the stability of this nonlinear parametric oscillator. Collective dynamics for systems of ions confined in quadrupole 3D traps with cylindrical symmetry is characterized in [24]. Ion dynamics in a linear combined trap has been investigated both theoretically and experimentally in [25], and it was shown that the presence of a homogeneous magnetic field superimposed over the applied DC and RF electric fields, leads to a set of coupled Mathieu equations. Coherent states for a set of quadratic Hamiltonians in the trap regime are constructed in Ref. [26], and then particularized to the asymmetric Penning trap. A method of finding a set of generators that form a closed Lie algebra, which creates a framework to characterize a general quantum Hamiltonian is presented in [27]. Thus, the Lie algebra can be extended to study the Hamiltonian of a bi-dimensional charged particle in time-dependent electromagnetic fields, by exploiting the similarities between the terms of these two Hamiltonians.

This paper characterizes the evolution of squeezed coherent states of motion for ions confined in quadrupole and octupole combined and RF traps, using the coherent state formalism [17,20,28] developed in [19,29], and the time dependent variational principle (TDVP) [30,31]. We study the bosonic realization of the Lie algebra for the SU(1,1) group, and (generalized) coherent states in the Fock space for a trapped ion [32]. The paper is organized as follows: In Section 2 we propose a dequantization algorithm [33] that enables explicit computation of the quantum Hamilton function associated to an anharmonic oscillator (ion) confined in a combined or RF trap, which describes an algebraic model when the anharmonic part is a polynomial. Such model is linear for 3D quadrupole ion traps that exhibit axial symmetry. Section 2 presents the solutions of the Schrödinger equation and the quasienergy spectrum for the model we suggest. The method suggested in [20,34] is developed and particularized in Section 3 for both combined (Paul and Penning) and RF traps, considering a RF anharmonic electric potential. We build the classical Hamilton functions for such particular traps and find the classical Hamilton equations of motion for the anharmonic combined trap, which represents another original result. We suggest that in the pseudopotential approximation case (ideal RF trap), the points of minimum of the classical Hamiltonian describe equilibrium configurations for trapped ions, of interest for implementing quantum logic. The results are discussed in Section 4, as the model is straightforward to extend for 2D ion traps.

2. Quantum dynamics of ions in axially symmetric quadrupole and octupole ion traps

Algebraic models for axially symmetric nonlinear quadrupole and octupole traps. The dynamical group $Sp(2,\mathbb{R})_a \otimes Sp(2,\mathbb{R})_r \otimes SO(2)$ associated to the Hamilton function which describes the dynamics of an ion of mass M and electric charge Q, confined within a 3D RF ion trap that exhibits both cylindrical and reflection symmetry, is the direct product between the axial and radial symplectic groups, and the rotations group $SO(2)$ generated by the axial angular moment operator L_z [29]. The Lie algebra basis of $Sp(2,\mathbb{R})_j,\ j = a, r$, is spanned by the generators K_{0j}, K_{1j} and K_{2j}. We introduce the infinitesimal generators of the axial symplectic group $Sp(2,\mathbb{R})_a$ defined as [20,29]

$$K_{0,1a} = \frac{M\omega_a}{4\hbar}\left[z^2 \pm \frac{p_z^2}{M^2\omega_a^2}\right] \ , \ K_{2a} = \frac{i}{4\hbar}\left[2z\frac{\partial}{\partial z} + 1\right] \ , \tag{1}$$

where $\omega_a/2\pi$ is the trap axial frequency. The commuting relations are

$$[K_{0a}, K_{1a}] = iK_{2a} \ , \ [K_{2a}, K_{0a}] = iK_{1a} \ , \ [K_{1a}, K_{2a}] = iK_{0a} \ . \tag{2}$$

Using Eqs. (1) we infer

$$(K_{0a} + K_{1a})\,(K_{0a} - K_{1a}) = iK_{2a} + \frac{1}{4\hbar^2}z^2p_z^2 \ . \tag{3}$$