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Band-gap structure and chiral discrete solitons in optical lattices with artificial gauge fields

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A B S T R A C T

We study three-leg-ladder optical lattices loaded with repulsive atomic Bose–Einstein condensates and subjected to artificial gauge fields. By employing the plane-wave analysis and variational approach, we analyze the band-gap structure of the energy spectrum and reveal the exotic swallow-tail loop structures in the energy-level anti-crossing regions due to an interplay between the atom–atom interaction and artificial gauge field. In further, we discover stable discrete solitons residing in the forbidden zone above the maxima of the highest band, and we find that these discrete solitons are associated with chiral edge currents.

1. Introduction

It is of great interest in realizing gauge potentials for neutral atoms [1–4], which may open a way to mimic the effects of magnetic fields acting on charged particles via neutral atomic systems. The gauge potentials bring nontrivial characters for single-particle dispersion and induce several novel quantum phenomena from single- to many-body levels. For instance, the spin–orbital (SO)

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coupling may give rise to double-well dispersion structure and Dirac points, which lead to anisotropic critical superfluid velocity [5], zitterbewegung effects [6,7] and semiclassical spin Hall effect [8]. Taking into account the atom–atom interactions in the SO coupled Bose–Einstein condensates (BECs), there appear several macroscopic quantum phenomena including vortices [9,10], monopoles [11], domains [12], skyrmions [13], and solitons [14,15].

More recently, the artificial gauge fields in optical lattices have attracted extensive attentions. The effective magnetic fields in optical lattices have been achieved by laser-assisted tunneling [16–19] and dynamical shaking [20,21]. In addition, the artificial gauge fields have also been created in optical superlattices [22] and in optical lattices with synthetic dimension [23–25]. These lattice systems with artificial gauge fields are well suitable for exploring the Hofstadter butterfly [26], quantum Hall states [27], topological insulators [28] and topological phase transitions [29–31]. In particular, it has been demonstrated that the gauge fields may induce exotic chiral currents [22] and chiral edge states [24,25].

Due to the atom–atom interaction, these systems provide a promising platform to explore the interplay between gauge field, nonlinearity and lattice potential. It has been demonstrated that, due to the interplay between nonlinearity and lattice potential, exotic swallow-tail loops appear in the band-gap structures [32–36]. Moreover, due to the balance between nonlinearity and dispersion, spatially localized nonlinear modes can exist in the forbidden gap [37–39]. In the last three years, many interesting macroscopic quantum phenomena of BECs in optical lattices subjected to artificial gauge fields have been explored. Lots of stable gap and gap-stripe solitons in SO coupled BECs in spin-dependent optical lattices, which can be classified according to physical symmetries, have been found [40,41]. Based upon a generalized discrete nonlinear Schrödinger equation including various types of SO couplings, it has been demonstrated the existence of discrete solitons with different miscibilities [42] and symmetries [43]. However, there are still many important open questions, such as, (1) How the interplay between the gauge field and the nonlinearity affects the band-gap structure? and (2) Are there stable discrete solitons in the energy gap?

In this paper, we consider atomic BECs in a three-leg-ladder model with an artificial magnetic flux and study their band-gap structures and discrete solitons. We find that, due to an interplay between the nonlinear atom–atom interaction and artificial gauge fields, swallow-tail loops appear in the band-gap structures. In addition, we numerically demonstrate the existence of stable discrete solitons above the maxima of the highest energy band. In a contrast to the solitons studied earlier, the discrete solitons in our system show chiral edge currents. Moreover, different from the uniform chiral edge currents in a non-interacting system, the chiral edge currents in our system are spatially localized in the discrete solitons. Additionally, we analyze stability of the discrete solitons by employing the linear stability analysis.

The paper is organized as following. In Section 2, we describe our model. Based upon the stationary extended states, we give the band-gap structures for different gauge fields and interaction strengths in Section 3. In Section 4, we study stable discrete solitons and discuss their properties. Finally, we summarize our results in Section 5.

2. Model

We consider atomic BECs held in a three-leg-ladder subjected to an artificial uniform magnetic field, see Fig. 1. In experiments, the three-leg-ladder can be realized by cutting a two-dimensional optical lattice via an one-dimensional box potential [44], and the artificial magnetic field can be created by using laser-assisted tunneling [22]. The system obeys the tight-binding Hamiltonian:

$$
\begin{aligned}
H = & -J\sum_{l=1}^{L_x}\sum_{\alpha=-1}^{1}(e^{-i\alpha\phi}\hat{b}^{\dagger}_{l+1,\alpha}\hat{b}_{l,\alpha}+\mathrm{H.c.}) - K\sum_{l=1}^{L_x}\sum_{\alpha=0}^{1}(\hat{b}^{\dagger}_{l,\alpha-1}\hat{b}_{l,\alpha}+\mathrm{H.c.}) \\
& +\frac{g}{2}\sum_{l=1}^{L_x}\sum_{\alpha=-1}^{1}\hat{n}_{l,\alpha}(\hat{n}_{l,\alpha}-1).
\end{aligned} \tag{1}
$$

Here, $\hat{b}_{l,\alpha}$ annihilates a boson on the site (l,α), $\hat{n}_{l,\alpha}=\hat{b}^{\dagger}_{l,\alpha}\hat{b}_{l,\alpha}$ is the particle number operator, g is the on-site interaction strength, J and K are respectively the intra- and inter-chain hopping strengths, and