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The harmonic oscillator in a space with a screw dislocation

Paolo Amore [a], Francisco M. Fernández [b,*]

[a] Facultad de Ciencias, CUICBAS, Universidad de Colima, Bernal Díaz del Castillo 340, Colima, Colima, Mexico

[b] INIFTA (CONICET), División Química Teórica, Blvd. 113 y 64 (S/N), Sucursal 4, Casilla de Correo 16, 1900 La Plata, Argentina

HIGHLIGHTS

- Quantum-mechanical systems with screw dislocation are receiving considerable interest.
- The harmonic oscillator with screw dislocation is not completely separable.
- The screw dislocation removes the degeneracy.
- The variational method is useful for obtaining eigenvalues and eigenfunctions.
- The spectrum exhibits a rich structure of avoided crossings.

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ABSTRACT

We obtain the eigenvalues of the harmonic oscillator in a space with a screw dislocation. By means of a suitable nonorthogonal basis set with variational parameters we obtain sufficiently accurate eigenvalues for an arbitrary range of values of the space-deformation parameter. The energies exhibit a rich structure of avoided crossings in terms of such model parameter.

1. Introduction

Space dislocations have been useful for the description of a variety of physical phenomena. Among such applications we mention an analysis of the influence of frozen-in topological defects in a crystal on the long-wavelength quantum states of a particle [1], a study of electrons moving in a magnetic

* Corresponding author.
E-mail addresses: paolo@ucol.mx (P. Amore), fernande@quimica.unlp.edu.ar (F.M. Fernández).

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field in the presence of a screw dislocation [2], the scattering of electrons on a screw dislocation [3], an investigation of the quantum scattering of an electron by a screw dislocation with an internal magnetic flux [4], a geometric model for the explanation of the origin of the observed shallow levels in semiconductors threaded by a dislocation density [5], the influence of the Aharonov–Casher effect on the Dirac oscillator in three different scenarios of general relativity [6], an investigation of torsion and noninertial effects on a spin-1/2 quantum particle in the nonrelativistic limit of the Dirac equation [7], a study of ac electronic transport in semiconductor crystals with a screw dislocation [8], a two-dimensional electron gas on a cylindrical surface with a screw dislocation [9], a study of spin currents induced by topological screw dislocation and cosmic dispiration [10], an analysis of a relativistic scalar particle with a position-dependent mass in a spacetime with a space-like dislocation [11], a study of the influence of a screw dislocation on the energy levels and the wavefunctions of an electron confined in a two-dimensional pseudoharmonic quantum dot under the influence of an external magnetic field and an Aharonov–Bohm field [12] and the effect of a screw dislocation on an anharmonic oscillator [13].

The present paper is motivated by those of Filgueiras et al. [12] and Bakke [13] who solve the Schrödinger equation with a screw dislocation. In the former case the authors choose a deformed potential $V_d(\rho)$ and a scalar pseudoharmonic interaction $V_{\mathrm{conf}}(\rho)$, $\rho^2 = x^2 + y^2$, so that the motion of the electron along the z-axis is free. Under these conditions the resulting eigenvalue equation is separable and exactly solvable. Later the authors consider that the electrons are confined by infinite walls at $z = 0$ and $z = d$ and claim that the eigenvalue equation is still separable. In the latter case the author chooses a potential $V(\rho)$ so that the motion of the particle is unbounded along the z direction and the spectrum continuous. Here we choose one of the simplest confining potentials, the three-dimensional harmonic oscillator, and obtain approximate eigenvalues of the resulting nonseparable deformed Schrödinger equation. In Section 2 we develop the main equations for the model, in Section 3 we first obtain approximate results by means of a simple variational ansatz that later use as the starting point of a more accurate Rayleigh–Ritz variational calculation. In this section we show results for different quantum numbers in a range of values of the dislocation parameter. Finally, in Section 4 we summarize the main results and draw conclusions.

2. The model

Some kind of topological defects are described by means of the line element

$$ds^2 = g_{ij} dy^i dy^j, \tag{1}$$

where g_{ij} are the elements of the metric tensor, $\{y^i\}$ is a suitable set of curvilinear coordinates and summation on repeated indices is assumed. The Laplacian in such a space is given by

$$\nabla^2 = \frac{1}{\sqrt{|\mathbf{g}|}} \partial_i \sqrt{|\mathbf{g}|} g^{ij} \partial_j, \tag{2}$$

where $|\mathbf{g}|$ is the determinant of the matrix $\mathbf{g} = (g_{ij})$, $\partial_i = \frac{\partial}{\partial y^i}$ and $g^{ij} g_{jk} = \delta^i_k$. Valanis and Panoskaltsis [14] derive expressions for a wide variety of deformations in a material body.

The Hamiltonian operator for a particle of mass m moving in such a space under the effect of a potential-energy function $V(\mathbf{r})$ is

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}). \tag{3}$$

In order to solve the Schrödinger equation for H it is convenient to choose a convenient set of units. We choose a set of dimensionless coordinates $\mathbf{r}' = \mathbf{r}/L$, where L is an arbitrary length, and rewrite the Hamiltonian operator in dimensionless form as

$$\frac{2mL^2}{\hbar^2} H = \nabla'^2 + v(\mathbf{r}'), \;\; v(\mathbf{r}') = \frac{2mL^2}{\hbar^2} V(L\mathbf{r}'), \tag{4}$$

where $\nabla'^2 = L^2 \nabla^2$