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Radiative gravitational collapse to spherical, toroidal and higher genus black holes

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ABSTRACT

We derive the matching conditions between FLRW and generalised Vaidya spacetimes with spherical, planar or hyperbolic symmetry, across timelike hypersurfaces. We then construct new models of gravitational collapse of FLRW spacetimes with a negative cosmological constant having electromagnetic radiation in the exterior. The final state of the collapse are asymptotically AdS black holes with spherical, toroidal or higher genus topologies. We analyse the collapse dynamics including trapped surface formation, for various examples.

1. Introduction

Modelling the process of gravitational collapse to black holes has been an important challenge in General Relativity and, ever since the first model of Oppenheimer and Snyder [1], huge progress has been made. However, many attempts to build such models using non-spherical exact solutions of the Einstein field equations (EFEs) have found no-go results, see e.g. [2] for a review.

The Oppenheimer–Snyder model results from the matching of a collapsing (spatially homogeneous and isotropic) spherically symmetric Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime to a (static vacuum) Schwarzschild exterior. As a result of the matching conditions, the interior must be a dust fluid and, consequently, the collapse is continuous to a singularity. Generalisations of this model to spacetimes with a cosmological constant Λ have also, more recently, been constructed [3].

The inclusion of the cosmological constant inspired the study of models of collapse to the so-called *topological black holes* [4–6]. This term has been used to coin black holes with topologies different from the sphere, e.g. with toroidal and higher genus topologies. Indeed, a variety of models of gravitational

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collapse to topological black holes have been constructed with Λ-vacuum exteriors, and interiors given by FLRW [4,5] and inhomogeneous spacetimes [7].

None of the above mentioned models includes radiation in the exterior. In order to do that using exact solutions of the EFEs, one can use generalisations of the (radiating) Vaidya metric, such as the Robinson–Trautman spacetimes, which result from the coupling of the EFEs to the Maxwell equations in vacuum and can include a cosmological constant as well as the different spatial topologies (see e.g. [8]).

For the particular case of Vaidya exteriors (which are spherically symmetric and have $\Lambda = 0$), some interiors are well-known, such as the FLRW solutions, studied in detail in [9], and the (inhomogeneous dust) Lemaître–Tolman solutions, investigated in [10,11].

Given the importance of the cosmological constant to modern cosmology and the possibility of the existence of a "landscape" of vacua states in string theory with Λ positive, negative and zero, it would be interesting to consider radiating exteriors with a cosmological constant (of any sign) within a class of generalised Vaidya spacetimes, which are sub-classes of the Robinson–Trautman spacetimes.

The global structure of Robinson–Trautman spacetimes with Λ has been studied in [12], where it was observed that these spacetimes may be used as exact models of black hole formation in non-spherical settings which are not asymptotically flat. In fact, the gravitational collapse of Robinson–Trautman spacetimes with toroidal topology has very recently been analysed in [13], where it was shown that incoming electromagnetic radiation can form black holes as well as naked singularities. However, as far as we know, the existence of interiors to such spacetimes has not been studied yet.

In this paper, we intend to construct models of collapse to black holes by investigating appropriate interiors to sub-classes of Robinson–Trautman spacetimes. The resulting models, therefore, would represent the gravitational collapse of an astrophysical object with a radiating exterior. The black hole would have spherical, toroidal or higher genus topology depending on the topology of the collapsing model.

The problem of matching two spacetimes is non-trivial, however, even for FLRW spacetimes. In fact, it is known that some matchings with FLRW are impossible: for instance, with a static vacuum [2,14] or with Einstein–Rosen (gravitational wave) exteriors [15,16], in cylindrical symmetry. Indeed, one of the issues has been to ensure that the matching problem can be solved globally in time, e.g., in some situations, ensuring that the matching surface maintains its character during the collapse and that e.g. trapped surfaces, which appear in the interior, will eventually match exterior ones. Even if the matching is possible, another problem that may arise is that one may not be able to construct it from initially untrapped surfaces (see an example in [16]).

All examples mentioned in the previous paragraph have vacuum exteriors and, in that case, due to the matching conditions, the interior radial pressure has to vanish at the matching boundary thus simplifying the problem. For example, if the interior is FLRW and the exterior has vacuum then, since the interior has to be dust, this implies that the boundary is ruled by geodesics which can considerably simplify the mathematics.

By considering Robinson–Trautman exteriors though, one faces a more difficult problem since not only the exterior is dynamical, but it also contains radial pressure. The interior, if FLRW, also needs to contain radial pressure, which brings extra degrees of freedom into the problem. Furthermore, while for Λ-dust matter it is possible to obtain explicit solutions from Friedmann's equations in terms of elementary functions, this is not the case, in general, for FLRW with pressure. In fact, for the hyperbolic FLRW case with $\Lambda \neq 0$, explicit solutions are only known for few particular (linear) equations of state, such as radiation fluids [8,17,18]. So, the analysis of the matching problem can also be hampered by that fact.

The plan of the paper is the following: In Section 2, we briefly revise the spacetimes to be matched, which are the FLRW and the generalised Vaidya spacetimes, both with a cosmological constant and with the different possible symmetries. In Section 3, we consider the matching between the two spacetimes, along timelike surfaces of symmetry, and derive the general matching conditions. In Section 4, we restrict the problem to $\Lambda < 0$, which ensures black hole formation and specialise the interior to have a linear equation of state. We then analyse the matched spacetimes for all possible cases with spherical topology and for some particular cases with toroidal and higher genus topologies, all satisfying the weak energy conditions. Finally, Section 5 contains our conclusions.

We use units such that $8\pi G = c = 1$, greek indices $\alpha, \beta, \ldots, \mu, \nu.. = 0, 1, 2, 3$, latin indices $a, b, c, ..i, j, k.. = 1, 2, 3$ and capitals $A, B, .. = 1, 2$.