An even simpler understanding of quantum weak values

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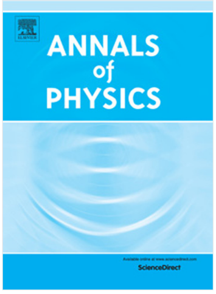

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An even simpler understanding of quantum weak values

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ABSTRACT:
We explain the properties and clarify the meaning of quantum weak values using only the basic notions of elementary quantum mechanics.

Keywords: quantum interference, uncertainty principle, weak measurements.

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> ...And look not for answers where no answers can be found.
>
> Bob Dylan

I. INTRODUCTION

In a recent publication [1] Qin and co-authors sought to provide a simplified understanding of the physics of the so-called weak measurements (for a recent review see [2]). They formulated their discussion in the framework of the quantum Bayesian approach [3], and followed other authors [4], [5] in asserting that "anomalous" weak values (WV) may not occur in a purely classical context. One may wonder whether a yet more straightforward explanation of these properties could be obtained directly from the basic principles of quantum theory. In the following, we will provide such an explanation.

II. PROBABILITY AND PROBABILITY AMPLITUDES

In quantum mechanics, e.g., in its field and many-body versions, the quantity of interest is often the probability $P^{\phi\leftarrow\psi}$ for the system to start in an initial state ψ and end up, after some time, in a final state ϕ. The resulting probabilities obey all the rules of the classical probability theory, but the quantum nature of the problem dictates that in order to evaluate $P^{\phi\leftarrow\psi}$, one must first obtain a complex valued transition *probability amplitude* $A^{\phi\leftarrow\psi}$ [6], so that

$$P^{\phi\leftarrow\psi}=|A^{\phi\leftarrow\psi}|^2. \tag{1}$$

Typically, an amplitude can be decomposed into various sub-amplitudes, corresponding to elementary processes, which all lead to the same outcome ϕ,

$$A^{\phi\leftarrow\psi}=\sum_n A_n^{\phi\leftarrow\psi}. \tag{2}$$

For example, for a system of interacting particles, $A_n^{\phi\leftarrow\psi}$ could correspond to Feynman diagrams describing various scattering scenarios [7]. The scenarios are "virtual", in the sense that only the probability amplitudes, and not the probabilities, can be ascribed to them individually. Together, virtual scenarios form a "real" pathway, connecting ψ with ϕ, which the system will be seen as taking with the probability (1), if the experiment is repeated many times.

III. THE DOUBLE-SLIT EXPERIMENT AND THE UNCERTAINTY PRINCIPLE

A simple illustration of the above is the Young's double slit experiment, sketched in Fig.1a. An electron starts at some location (x,y), and ends up in a final position (x',y'), which it can reach through two holes made in the screen. There are two virtual pathways, passing through the holes 1 and 2, with the probability amplitudes $A_1^{(x',y')\leftarrow(x,y)}$ and $A_2^{(x',y')\leftarrow(x,y)}$, respectively. A well known feature of quantum description is the impossibility do decide which of the two routes was actually taken. Any attempt to accurately determine it, destroys the interference pattern, by changing the probability $P^{(x',y')\leftarrow(x,y)}$ from $|A_1^{(x',y')\leftarrow(x,y)}+A_2^{(x',y')\leftarrow(x,y)}|^2$ to $|A_1^{(x',y')\leftarrow(x,y)}|^2+|A_2^{(x',y')\leftarrow(x,y)}|^2$. If no such attempt is made, "one may *not* say that an electron goes either through hole 1 or hole 2" [6]. The two virtual routes together form for the electron a single real pathway from (x,y) to (x',y'). This is the *uncertainty principle* [6].

A further simplification of the double slit experiment, which brings us closer to issue of weak values , is shown in Fig. 1b. Let a system, consisting of spin 1/2, start in a state $|\psi\rangle$ at $t=0$, evolve with a Hamiltonian $\hat{H}$ until $t=T$, and then be observed in the final state $|\phi\rangle$. Choosing an arbitrary basis $\{|i\rangle\}$, $\langle i|j\rangle=\delta_{i,j}$, $i=1,2$, and inserting the unity $\sum_{i=1}^{2}|i\rangle\langle i|$ at $t=T/2$, we can write