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Unified theory of effective interaction

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ABSTRACT

We present a unified description of effective interaction theories in both algebraic and graphic representations. In our previous work, we have presented the Rayleigh–Schrödinger and Bloch perturbation theories in a unified fashion by introducing the main frame expansion of the effective interaction. In this work, we start also from the main frame expansion, and present various nonperturbative theories in a coherent manner, which include generalizations of the Brandow, Brillouin–Wigner, and Bloch–Horowitz theories on the formal side, and the extended Krenciglowa–Kuo and the extended Lee–Suzuki methods on the practical side. We thus establish a coherent and comprehensive description of both perturbative and nonperturbative theories on the basis of the main frame expansion.

1. Introduction

The concept of the effective interaction is, and will continue, playing a very important role in understanding various quantum many-body systems [1–16]. We first divide a large Hilbert space into a model space (P-space) of a tractable size and its complement (Q-space). Then we define the effective interaction v in the P-space to describe a set of selected eigenstates of the full Hamiltonian H.

The effective interaction has been formulated in both perturbative and nonperturbative ways. The Rayleigh–Schrödinger (RS) [17] and Bloch [7–9] theories describe the effective interaction v in perturbative ways, and their interrelation has been understood clearly by introducing the main frame expansion [18]. On the other hand, nonperturbative description of the effective interaction is far from being satisfactory, as shown by the "current situation" in Fig. 1. On the formal side, the Bloch–Horowitz (BH) [19,20] and Brillouin–Wigner (BW) [21] theories are limited essentially to one-dimensional

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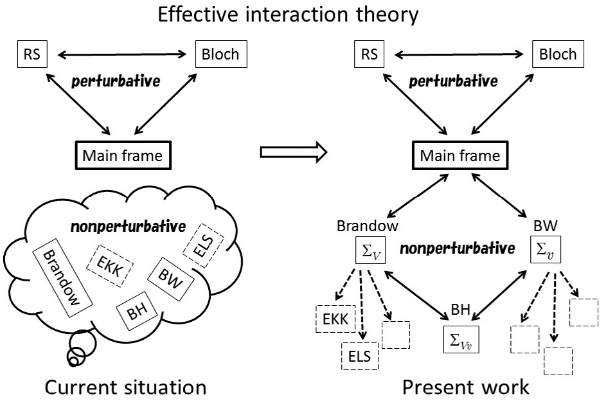

Fig. 1. Pictorial explanation of the present work. Left: current situation of our understanding of the effective interaction. Relations among various nonperturbative theories are hidden in the cloud, while the perturbative sector has been well understood via the main frame expansion [18]. Right: the present work establishes a unified description of both perturbative and nonperturbative theories on the basis of the main frame expansion. The Brandow, Brillouin–Wigner (BW), and Bloch–Horowitz (BH) theories are replaced by their generalizations $\sum_V$, $\sum_v$, and $\sum_{Vv}$ formulas, respectively. They provide various recursive formulas indicated by dashed lines, which include the extended Krenciglowa–Kuo (EKK) and the extended Lee–Suzuki (ELS) methods.

P-space, and the Brandow theory [1–3] requires degenerate P-space.[1] On the practical side, the extended Krenciglowa–Kuo (EKK) and the extended Lee–Suzuki (ELS) methods are now available to calculate v in general nondegenerate P-space [10,11,13]. However, their relation to the above formal theories has been left unclear especially in nondegenerate P-space.

In this uncomfortable situation, we start from the main frame expansion, and derive various nonperturbative expressions for v in a unified fashion in both algebraic and graphic representations in general nondegenerate P-space. In the formal aspect, our approach reveals the interrelation not only among nonperturbative theories, but also between perturbative and nonperturbative theories as shown by the "present work" in Fig. 1. In the practical aspect, the present work provides basic formulas which lead to various recursive methods to calculate v.

The plan of this paper is the following. In Section 2, we explain the effective interaction v fixing the notation. In Section 3, we introduce the effective transition potential u between P- and Q-spaces. In Section 4, we explain the main frame expansion of u in the bracketing and folded diagram representations.[2] In Section 5, we survey the nonperturbative theories of v and their interrelation we are going to establish in the following sections. In Sections 6 and 7, we start from the main frame expansion, and derive two identities which we refer to as the $\sum_V$ and $\sum_v$ formulas. They are conveniently expressed in the generalized bracketing and generalized folded diagram representations.[3] We shall see that these identities are generalizations of the Brandow and BW theories to nondegenerate multi-dimensional P-space, as indicated in the "present work" in Fig. 1. In Section 8, we derive the $\sum_{Vv}$ formula both from the $\sum_V$ and $\sum_v$ formulas, which is shown to be a multi-dimensional generalization of the BH theory. In Section 9, using the above results for u, we explain the effective interaction v in nonperturbative theories putting emphasis on the practical aspect. We shall see that the $\sum_V$ formula yields the EKK and ELS methods [10,11] directly. Furthermore, we demonstrate that the $\sum_V$ and $\sum_v$ formulas lead to plenty of recursive methods which remain still to be investigated. In Section 10, we give intuitive explanations of the above nonperturbative theories using one-dimensional P-space.

1 As is well known, the usual "BW perturbation theory" in one-dimensional P-space does not give an expansion of the true eigenenergy E in powers of the interaction V, but yields an equation to determine E self-consistently. In this work, therefore, we classify the "BW theory" as a nonperturbative approach.

2 Further explanation of the bracketing and folded diagram representations is presented in Appendix A.

3 These generalized representations are summarized in Appendix B.