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Logical inference approach to relativistic quantum mechanics: Derivation of the Klein–Gordon equation

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HIGHLIGHTS

- Logical inference applied to relativistic, massive, charged, and spinless particle experiments leads to the Klein–Gordon equation.
- The relativistic Hamilton–Jacobi is scrutinized by employing a field description for the four-velocity.
- Logical inference allows analysis of experiments with uncertainty in detection events and experimental conditions.

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ABSTRACT

The logical inference approach to quantum theory, proposed earlier De Raedt et al. (2014), is considered in a relativistic setting. It is shown that the Klein–Gordon equation for a massive, charged, and spinless particle derives from the combination of the requirements that the space–time data collected by probing the particle is obtained from the most robust experiment and that on average, the classical relativistic equation of motion of a particle holds.

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1. Introduction

The inception of quantum theory was one of taking leaps. This is illustrated by e.g. Schrödinger's paper [1] in which he proposes his celebrated wave equation. In this article [1] Hamilton's principal function S is postulated to take the form $S = k \ln \psi$ with k a constant and is then substituted in the Hamilton–Jacobi equation (HJE). Upon variation of the resulting quadratic functional with respect to ψ (which Schrödinger later justifies using Huygens' principle [2]) an equation linear in ψ, now known as the Schrödinger equation, was obtained. The derivation of the Klein–Gordon equation [3–7] is essentially identical to that of the Schrödinger equation namely, an action Ansatz is substituted in the relativistic Hamilton–Jacobi equation, and after variation of the resulting quadratic functional with respect to ψ, the relativistic analogue of the Schrödinger equation is obtained [3–7].

Because of the ad hoc assumptions involved in obtaining these equations, standard quantum mechanics textbooks usually present the formalism of quantum theory as a set of postulates (see e.g. Refs. [8–11]) and considerable activity focuses on eliminating some of these postulates [12–19]. Instead of starting from a set of postulates, the current work presents an alternative derivation of the relativistic wave equation based on the principles of logical inference (LI) [20–23]. Specifically, we demonstrate how the Klein–Gordon equation for a massive, charged and spinless particle follows from LI based on the analysis of data recorded by a detector, thereby extending earlier work [24–27] to the relativistic domain.

The key concept in LI is the plausibility [23], a mental construct which quantifies e.g. the chance that a detection event occurs. In general, the degree of plausibility is expressed by a real number in the range of 0 and 1 [23]. The algebra of LI facilitates plausible reasoning in the presence of uncertainty in a mathematically well-defined manner [20–23]. In real experiments there is not only uncertainty about the individual detection events but there obviously is also uncertainty in the conditions under which the experiments are carried out. Inevitably, the conditions of the experiment will vary whenever the experiment is repeated. But if the experimental data is to be reproducible, the experiment must be robust (to be quantified later) with respect to small changes in the conditions under which the experiment is being performed. Earlier work has shown that the equations of non-relativistic quantum theory can be obtained by analyzing such robust experiments [24–27]; most notable are the Schrödinger [25] and the Pauli equation [26]. Importantly, the requirement that the experiment is to be robust implies that the plausibility must be viewed as an objective assignment (i.e. conditional probability) rather that a subjective one [25]. The present work extends this approach to the relativistic domain: it shows how the Klein–Gordon equation [3,7] for a massive, charged, and spinless relativistic particle emerges by an analysis similar to the one employed in Refs. [25,26].

2. Logical inference approach

2.1. Particle detection experiment

Consider an experiment in which a particle source and detectors are located at fixed positions relative to the laboratory reference frame. The source emits a particle that interacts with one of the detectors and triggers a detection event that yields data in the form of three spatial coordinates $\mathbf{r} = (x, y, z)$ of the detector and the clock time t at which the event occurred. The experiment is considered to be ideal in the sense that every emitted particle triggers one and only one detector.

The experiment is repeated N times, meaning that we let N particles pass through the detector. Each time a particle is created, the (laboratory) clock time is reset. We label the particles and the corresponding data by the index $n = 1 \ldots N$ and denote the spatial and temporal resolution by Δ_s and Δ_t, respectively. As particle n passes through the detector, the latter produces a time stamp t_n and a vector of spatial coordinates $\mathbf{r}_n = (x_n, y_n, z_n)$, which because of the limited resolution, correspond to the time-bin $j_n = \text{ceiling}(t_n/\Delta_t)$ and space-bin $\mathbf{k}_n = \text{ceiling}(\mathbf{r}_n/\Delta_s)$ where, element-wise, the function ceiling(x) returns the smallest integer not smaller than x. In practice the number of time-bins and space-bins is necessarily finite. Therefore we must have $0 \le j_n \le J$ and $(0, 0, 0) \le \mathbf{k}_n \le \mathbf{K} = (K_x, K_y, K_z)$, where J, K_x, K_y and K_z are (large) integer numbers.