Annals of Physics 372 (2016) 283–296

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Algebraic approach and coherent states for a relativistic quantum particle in cosmic string spacetime

M. Salazar-Ramírez [a,*], D. Ojeda-Guillén [a], R.D. Mota [b]

[a] *Escuela Superior de Cómputo, Instituto Politécnico Nacional, Av. Juan de Dios Bátiz esq. Av. Miguel Othón de Mendizábal, Col. Lindavista, Del. Gustavo A. Madero, C.P. 07738, Ciudad de México, Mexico*

[b] *Escuela Superior de Ingeniería Mecánica y Eléctrica, Unidad Culhuacán, Instituto Politécnico Nacional, Av. Santa Ana No. 1000, Col. San Francisco Culhuacán, Del. Coyoacán, C.P. 04430, Ciudad de México, Mexico*

ARTICLE INFO

Article history:
Received 2 March 2016
Accepted 20 May 2016
Available online 26 May 2016

Keywords:
Algebraic methods
Coherent states
Cosmic string spacetime
Schrödinger factorization
Tilting transformation

ABSTRACT

We study a relativistic quantum particle in cosmic string spacetime in the presence of a magnetic field and a Coulomb-type scalar potential. It is shown that the radial part of this problem possesses the $su(1,1)$ symmetry. We obtain the energy spectrum and eigenfunctions of this problem by using two algebraic methods: the Schrödinger factorization and the tilting transformation. Finally, we give the explicit form of the relativistic coherent states for this problem.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

The cosmic strings, which are hypothetical 1-dimensional topological defects were introduced in the 1970s [1]. In the 1980s and the early 90s a strong interest arose in connection with cosmic strings [2]. This interest is due to the bridge that these cosmic strings provide between the physics of the very small and the very large. It is known from the literature [3,4] that cosmic strings are predicted in some unified theories of particle interactions and could have formed in one of the numerous phase transitions in the early universe due to the Kibble mechanism. These cosmic strings could also be responsible for the large-scale structure of the universe.

* Corresponding author.
E-mail address: escomphysics@gmail.com (M. Salazar-Ramírez).

http://dx.doi.org/10.1016/j.aop.2016.05.011
0003-4916/© 2016 Elsevier Inc. All rights reserved.

The study of quantum systems under the influence of the gravitational field has been of great interest in particle physics. A problem that has been extensively studied is the appearance of topological phases in the quantum dynamics of a single particle moving freely in multiple connected space-times. As an example of a gravitational effect of topological origin we can consider the analogue of the electromagnetic Aharonov–Bohm effect [5,6]. This effect is provided when a particle is transported around an idealized cosmic string [7–10] along a closed curve in which case the string is noticed at all. The influence of the Aharonov–Casher effect [11] on the Dirac oscillator has been recently studied in the Minkowski spacetime, the cosmic string spacetime and the cosmic dislocation spacetime [12]. The Aharonov–Casher geometric phase in the presence of topological defects as the cosmic string spacetime and the cosmic dislocation spacetime is studied in Refs. [13,14]. Recently, the relativistic quantum motion of charged particles in the presence of a magnetic field and scalar potentials in the cosmic string spacetime has been studied [15,16]. More specifically, in Ref. [15] the authors considered the cases of relativistic spin-0 and spin-$\frac{1}{2}$ particles, where the energy spectrum and eigenfunctions are found in an analytical way. In this reference the vector potential is defined proportional to the square of the polar distance to the string. In Ref. [16], the author defined a different vector potential proportional to the polar distance to the string and used this to solve this problem in an algebraic way by using the Schrödinger factorization method and the unitary irreducible representation theory of the $SU(1,1)$ group theory.

The factorization method introduced by Infeld and Hull [17,18] has been of great importance in the study of quantum systems. In their work, Infeld and Hull gave a systematic method to factorize and classify a large class of potentials. However, Dirac [19] and Schrödinger [20–22] established the fundamental ideas of factorization in quantum physics. The importance of these factorization methods lies in the fact that if the Schrödinger equation is factorizable, the energy spectrum and eigenfunctions are obtained in an algebraic way. Moreover, the operators constructed from these methods are related to compact and non-compact Lie algebras.

Besides, the existence of a symmetry group has been used to compute the coherent states of many physical problems [23–27]. The harmonic oscillator coherent states were introduced by Schrödinger while he was looking for a system which possessed a classical behavior [28]. The importance of coherent states in quantum optics was studied in Refs. [29–32] and the generalization of these states to any algebra of a symmetry group is reported in Refs. [33–35].

The aim of the present work is to study a relativistic quantum particle in cosmic string spacetime in the presence of a magnetic field and scalar potential in an algebraic way. In order to obtain the energy spectrum and the eigenfunctions of this problem we use the theory of unitary representations and two algebraic methods: The Schrödinger factorization method and the tilting transformation. From the $SU(1,1)$ group theory we construct the relativistic coherent states for the radial part.

This work is organized as follows. In Section 2 we obtain the uncoupled second-order differential equations satisfied by the radial components. In Section 3, we apply the Schrödinger factorization method to one of the uncoupled equations to obtain the energy spectrum and the eigenfunctions of our problem. In Section 4, the energy spectrum and the eigenfunctions are obtained by using the tilting transformation and a realization of the $su(1,1)$ Lie algebra which is energy-independent. In Section 5 we obtain the explicit expression of $SU(1,1)$ Perelomov coherent states for a relativistic quantum particle in cosmic string spacetime in the presence of a magnetic field and scalar potential. Finally, we give some concluding remarks.

2. The Dirac equation for the cosmic string spacetime

The metric tensor for the cosmic string spacetime has been used in several problems as can be seen in Refs. [36–43]. In cylindrical coordinates this metric tensor is defined by the line element

$$ds^2 = dt^2 - dr^2 - \rho^2 r^2 d\phi^2 - dz^2, \tag{1}$$

where the coordinates $(t, z) \in (-\infty, \infty)$, $r \geq 0$, the angular variable $\phi \in [0, 2\pi]$ and $\rho = 1 - 4\mu$ is related to the deficit angle, with μ the linear mass density. Let us consider a magnetic field parallel

Download English Version:

https://daneshyari.com/en/article/8201707

Download Persian Version:

https://daneshyari.com/article/8201707

Daneshyari.com